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Optimized Performance of Multi-Level Pulsed Signaling with Photon Counting

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ABSTRACT

Optical communications systems using pulsed optical modulations and photon-counting receivers have been demonstrated to be capable of very high photon efficiencies. The basic trade is to achieve high photon efficiency by using more bandwidth along with higher ratios of peak-to-average transmitted power. We investigate here the efficiency a photon-counting receiver could achieve in the lower bandwidth regime. It is well known that the best efficiencies at high bandwidths are achieved using two-level pulsed signaling. We will show that it is possible to achieve medium efficiencies at somewhat reduced bandwidth if the pulses are allowed to have more than one level. Equivalently, we show that at fixed slot rates, it is possible to more than double the data rates achievable by a photon-counted, pulsed optical system, although at somewhat decreased photon efficiency.

Keywords: Photon-counting, signaling, receiver, PAM, PPM, PSK, efficiency, capacity, optical, communications.

1. INTRODUCTION

We often use the photon efficiency achievable at the receiver as a major design metric for assessing free-space laser communications systems. Especially when the system is such that the transmitter needs to be relatively large and high-power, or when the receive aperture must grow to be quite large, even a few decibels of efficiency variation could mean large swings in transmitter or receiver size (and cost.) Deep Space lasercom systems, in particular, are demanding designs requiring capable terminals in space and very large terrestrial apertures if they want to deliver appreciable data rates. Some recent and ongoing space-to-ground lasercom systems have thus turned to pulsed signaling formats that are receivable using photon-counting detection. (e.g. [1] - [7].) This pairing is known to offer extremely good photon efficiency, thus allowing, for example, the ground aperture to be relatively small.

A second major benefit of such a receiver is that it can operate through the turbulent atmosphere. Since photon-counting is a kind of energy detection that does not require all its spatial components to be in-phase, such receivers can be placed behind even large telescopes without incorporating turbulence mitigation techniques such as Adaptive Optics. (See Boroson[8] for a discussion of the noise penalties due to photon-counting in the daytime.)

One drawback of such approaches is the fact that the best system efficiency requires appreciable bandwidth expansion, in that the rate of the slots (some of which include pulses) is quite a bit higher than the source bit rate. The generation of such wide band signaling is straightforward out to tens of gigahertz using modern technologies. However, present-day photon-counting technologies are limited to slot rates of 5-10 gigahertz (although such limits will almost certainly be improved in the near future.)

In any case, as the community's desired data rates increase, there will usually be some technological (component bandwidth or peak transmitter power) limitations constraining the efficiency achievable. Thus, as the data rates go up, transmitter power and receiver aperture sizes will need to grow - first, because the desired data rate is increasing and second, because the achievable efficiency will be less good.

It is known that very high data rates can be transmitted and received with *pretty good* photon efficiency by using coherent techniques (and the fiber telecom community does exactly that.) Unfortunately, coherent systems require transmitter and receiver architectures that are quite different from photon-counting ones. That is, their modulators are quite different and their receivers need to work hard to undo the deleterious effects of the turbulent atmosphere, especially behind large telescopes. Thus, a system designer with a range of requirements would very likely like to choose one transmitter-receiver approach or the other.

We wonder, then, if we can try to share our photon-counting receiver between two classes of users. That is, can we build a single receiver structure to be used by both those who are forced to accept lower data rates because of, say, the link loss, and so want an efficiency as high as possible; and those who are able to deliver more optical power to the receiver and so would like to achieve as high a data rate as possible. For instance, a system optimized for lower-rate, higher-efficiency operation between Jupiter and the Earth will also spend much of its voyage at distances closer than that. We ask here whether such a system could support higher data rates when at shorter ranges (with lower transmission losses) without having to make major changes to the ground receiver (and making the transmitter support, in addition, phase-modulation modes) in order to support coherent formats.

The paper is thus organized as follows: we briefly review the channel capacity metric and use it to compare coherent and photon-counted formats; we present an overview of previous multi-level pulsed approaches; we develop a design optimization approach and give its results; we briefly discuss noisy systems; and we put this work in context with other signaling approaches.

2. CHANNEL MODEL

We model the channel as in Figure 1. Source bits are encoded with a (near) capacity-achieving channel code at a code rate to be selected. (Channel bit interleaving may also be inserted at this point if there is channel fading or similar phenomena (eg [9].) The now encoded channel bits are then mapped using a modulation mapping format. (In some designs, the encoding and the mapping are combined into a single, more complex step.) This formatted data is then modulated onto an optical carrier, amplified, and transmitted through the optical system and the free-space channel. At the receiver, the signal (possibly along with background light) is captured by the optical systems and is then coupled to the detection and demodulation system. This may be via a single mode fiber or via a multiple-spatial mode receiver. If there is background noise, there may be spectral filtering before the detection mechanism. The detector measurements are then fed to a soft decision receiver which makes optimum use of them to first deduce signal timing and then to produce demodulated and decoded bits as its best estimate of the original source bits.

Thanks to our use of channel coding, and with knowledge that near-capacity-achieving codes are available, we use Channel Capacity as the figure of merit. (We have used this approach to great advantage in many recent systems[10] , and have discussed its analysis with the photon-counting receiver[11].) Capacity, C, will be parameterized as source bits per channel symbol, and received power will be parameterized as average photons per channel symbol, N_s. Thus, N_s / C is the value, photons per source bit, which is the cost or, equivalently, (the inverse of) efficiency. The extra bandwidth required by such a system is the product of the inverse of C times the bandwidth expansion of the modulation mapping.

3. PULSED AND PHOTON-COUNTED OPTICAL MODULATIONS

The very best photon-counting detectors have nearly unity detection efficiency and additive noise rates much lower than the desired received flux. Thus, in the absence of background light, we can model noiseless photon-counted detection of received light as a Poisson process with an instantaneous rate proportional to the received flux. For time-slotted transmissions, then, detecting one or more photons in an "on" slot gives the correct response, and detecting no photons gives either the correct value for an empty slot or an erasure for an "on" slot. This erasure probability is thus

$\exp(-N_p)$ where we let N_p be the Poisson average detected-photon value for an "on" pulse.

The most widely used pulsed format is known as Pulse Position Modulation (PPM) where K channel bits are mapped into the timing of one pulse out of a set of $M = 2^K$ (faster) time slots. (See, e.g., Hamkins[12] for an excellent overview.) It can be shown that the achievable information transfer (we will here call this the capacity) for M-PPM is

$$C = K\left(1 - \exp(-N_p)\right) \text{ source bits per M-ary channel symbol}$$

(or, equivalently, $\frac{K}{M}\left(1 - \exp(-N_p)\right)$ source bits per channel slot time.)

This is a 2-level signaling scheme ("off" and "on") with $N_p = N_s = MN_{avg}$ where N_{avg} is the average photons per slot-time, and we see that the number of photons per PPM channel symbol is the same as the number of photons in an "on" pulse (ie at the peak power.) We can plot the efficiency of M-PPM for M=4 and M=64 (see Figure 2; note that we plot efficiency versus the total bandwidth expansion due to both the error-correction code and the PPM modulation.) We have also plotted in Figure 2 the possible efficiencies achievable for all possible M, assuming we choose at each bandwidth expansion that M that achieves the best efficiency. We can see that (and as is well known,) at high bandwidth expansion, this system can achieve an efficiency of multiple (as much as K) source bits per detected photon.

As above, we can observe that coded PPM can be thought of as the concatenation of the rate $\left(1 - \exp(-N_p)\right)$ error-correction code plus the rate K/M PPM modulation code.

4. COHERENT SYSTEMS

Before we examine possible improvements to our photon-counted PPM system, we compare this performance with several other modulation/receiver pairs. In particular, we examine a few coherent systems, all of which are well known to the radio communities.

Coherent optical receiver systems are well modeled as operating in additive Gaussian noise. A low-noise optical pre-amplifier followed by a linear receiver gives the same performance as in well-studied radio systems. It is known that the Gaussian optical SNR value (a pretty good approximation for an optimized low-noise pre-amplifier) is equivalent to the average number of received photons per measurement (slot.) (This same performance can also be achieved with a true optical heterodyne receiver. See, e.g. [13].) It is important to note that either of these optical receivers operates on a single spatial mode, and so would likely need adaptive optics or some related technique if it were to be placed behind a large telescope receiving light coming down through the turbulent atmosphere.

It can be shown that the formula for the capacity of this binary optical PSK channel (assuming soft decision decoding) can be written

$$C = 1 - E_x \log_2\left(1 + \exp(-2\sqrt{2N_s}(x + \sqrt{2N_s})\right)$$

where the expectation is over the Gaussian random variable, x, with mean 0 and variance 1.

PSK can also be received in a homodyne receiver. The formula for this capacity takes the same form, but replaces N_s by $2N_s$ because, as is well known, there is half as much Gaussian noise in the homodyne receiver. (See, e.g. [14].)

We plot the efficiency of these two PSK receivers along with the photon counting PPM receiver in Figure 3. We can see that at high M for PPM (with its commensurate wide bandwidth), one can achieve an efficiency better than homodyne. However, at smaller bandwidths, both homodyne and heterodyne PSK can outperform photon-counted PPM.

5. GENERALIZED TWO-LEVEL SIGNALING WITH PHOTON-COUNTING

We first ask – can we improve the PPM curve? We start by examining other two-level pulsed formats.

It is known that the optimum two-level photon-counted format is one where there is a mapping code such that the channel is presented with slots seemingly independently chosen with probability p_1 for pulses and probability $(1-p_1)$ for empty slots (Shamai[15].) One can concatenate this mapping code with an error-correction code to achieve capacity. (We note that PPM with multiple "on" pulses per symbol can achieve performance between PPM and this optimum.)

In fact, Barron[16] has invented a family of such concatenated codes by mixing the "shaping" and mapping property with the error correction properties.

It can be shown that the capacity for such a format (we can call it Generalized On Off Keying, or merely OOK) when using a photon-counted receiver, can be written

$$C = H\left[p_1 \left(1 - \exp(-N_s / p_1)\right)\right] - p_1 H\left[\exp(-N_s / p_1)\right]$$

where the average symbol photon count (a symbol here being a single slot) is $N_s = p_1 N_p, N_p$ is the average count in an "on" pulse, and H(p) is the binary entropy function. In Fig 4 we show the efficiency at capacity for p_1=1/4 and 1/64, as well as the curve showing the best performance as we vary p_1 through all possible values.

In Fig 5 we next compare photon-counted PPM and OOK, along with coherent PSK. We see that, of these, OOK has the best efficiency except at the very narrowest bandwidths, where homodyne PSK can be better.

6. MULTI-LEVEL SIGNALING

Before we examine further improvements to the photon-counted family, we need to be fair and see if the coherent systems could have done even better by using higher alphabet size signaling. In fact, Shannon's original work gave us a simple formula for optimum modulation with coherent receivers (without specifying the modulation formats.) He showed that the capacity of such a channel can be written

$$C = \frac{1}{2}(1 + SNR) \quad \text{bits per dimension} \quad \text{(see e.g., [14])}$$

which means that a heterodyne (or pre-amplified) coherent optical receiver can achieve

$$C = (1 + N_s) \quad \text{bits per channel usage (e.g. time slot)}$$

and a homodyne system can achieve

$$C_{\text{hom}} = \frac{1}{2}(1 + 4N_s)$$

Let us now examine multiple phase levels in PSK. Of course, this is only relevant to the heterodyne (or coherent pre-amplified) receiver and not the homodyne one. It is well known that QPSK (4-PSK) performs like two simultaneous 2-PSK systems with independent noises and non-interfering signals. Thus, it has the same efficiency curve as 2-PSK but shifted left by a factor of two in bandwidth. Capacity for 3-PSK, 8-PSK and higher values are also well known.

Although there is no quadrature component in a homodyne receiver, we can try to send a multi-amplitude set of signals, shown in Figure 6a. This is traditionally called Pulse Amplitude Modulation (PAM). The formulas for its capacity can be found in, for example, [17]. (In the additive Gaussian noise receivers, we usually choose the amplitude levels to be equally spaced.) In Figure 7, we show the performance of many of these formats.

Returning to the heterodyne/pre-amplified system, we could also create amplitude levels as well as phase shifts. Such a hybrid is well known as Quadrature Amplitude Modulation (QAM.) Formulas for their capacities are also given in [17]. We do not show their performance here, but they are known to fall between the MPSK and Gaussian optimum curves.

We finally note that these multi-level formats fall a bit short of the Shannon limits. Luckily, shaping codes have been invented that can push coherent performance essentially all the way to the limit. (See, eg, [17] for a discussion of such approaches. We do not show their performance here.)

To summarize, we can see in Fig 7 that achievable multi-level MPSK and QAM coherent, and PAM homodyne optical formats can achieve better bandwidth-efficiency tradeoffs than the two-level photon counted formats at narrow bandwidths.

7. MULTI-LEVEL SIGNALING WITH PHOTON-COUNTING

Turning finally to our pulsed, photon-counting systems, we now first wonder – is there a bound, similar to the Shannon capacity formula, for photon-counting receivers with unconstrained modulations? A number of researchers have attacked this problem over the years[15, 18-21]. It turns out that the answers are different for the two cases where one constrains the peak power in addition to the average power versus the case of only constrained average power. As we are interested in practical solutions, we limit ourselves to the peak and average constrained case. Even here, though, the best general solutions to date give upper and lower bounds for unconstrained modulation capacity only at high peak and average values. Thus, in the tradeoff region we are interested in, we need to investigate specific designs.

Shamai[15] has shown that, for the case of constrained peak and average powers, the capacity-achieving photon-counted modulation format takes the form of a finite set of various amplitude pulses with particular probabilities; that is, a generalized PAM. Of course, PAM for photon-counting cannot include 180-degree phase shifts as do the PAM designs for homodyne, and so we expect them not to have as much an efficiency gain as those. (See Fig 6b.)

The format is thus to send either “on” with two or more possible values or “off.” Furthermore, since Poisson statistics do not give symmetric error histograms near each possible “on” value, we need to find not only the capacity-optimum levels, but also the optimum probabilities for selecting each level. (This is analogous to the Shaping Code step in the Gaussian channels, which there was done separately.)

Multi-level pulsed optical signaling has been investigated over the years by a number of researchers, with one of the earliest being Hisdal[22] who used equi-spaced and equi-probable power levels. Some other more recent approaches include Haddad and Bross[23] who examined multi-level settings with PPM pulsing formats, and Lanka et al[24] who examined 4-ary PAM with quadratic spacing.

Capacity, assuming a Poisson counting model for such a modulation, can be shown to be

$$C_{PAM}(\{p_i, n_{s,i}\}, n_0, L) = -\sum_{k=0}^{\infty}\sum_{i=0}^{L-1} p_i \frac{(n_{s,i}+n_0)^k}{k!} e^{-(n_{s,i}+n_0)} \log_2 \sum_{l=0}^{L-1} p_l \left[\frac{n_{s,l}+n_0}{n_{s,i}+n_0}\right]^k e^{-(n_{s,l}-n_{s,i})}$$

where we let the L signal levels be $n_{s,0} = 0 < n_{s,1} < n_{s,2} < .... < n_{s,L-1}$ (where these terms are in average detected signal photons per timeslot), the set $\{p_0, p_1, p_2, ..., p_{L-1}\}$ are the probabilities (non-negative, summing to one) that these appear (in the shaping code), and, for completeness, we include n_0 as an additive, Poisson noise term, (which is zero in our noiseless case.) We see that the average signal count is

$$n_{avg} \equiv \sum_{l=0}^{L-1} p_l n_{si} \ ,$$

.

and the peak to average ratio is $M = n_{s,L-1} / n_{avg}$ where we use M as an analogy with PPM. (Of course, we can choose a non-integer M here.) We can note that the levels for photon-counted PAM are in terms of power (actually, energy, since we are suppressing explicit references to the slot time, T,) whereas for coherent systems, the levels are usually defined in terms of the amplitudes.

We have opted to investigate L=2, 3, and 4 for selected peak to average ratios, and we have performed a search over the pulse levels and probabilities to optimize capacity. We present the results in Fig 8 for the case of M=4 with varying average power. In 8a and 8b, the L=3 case, we can see how the optimum $\{p_1, p_2\}$ and $\{n_{s,1}, n_{s,2}\}$ vary with average power. The larger pulse appears in the shaping code around 18-20% of the time, and the smaller pulse occurs more often. In Figure 8c, we show the efficiency of optimized L=2, 3, and 4, as well as the particular L=3 design marked by arrows in (a,b). We see that, for M=4, the L=3 set is optimum between about 0.7 and 1.4 slots per source bit. Below 0.7, the L=4 set does better. We also see that the specific fixed design is optimum around 2 dB signal photons per slot. (The chart also shows coherent (heterodyne) 4PSK for comparison.) We observe that photon-counting efficiency is lost quickly as the PAM curves goes up to the left.

We have also optimized these L values for various M up to 8. We found that in this efficiency-bandwidth tradeoff region, there are only small differences between the different selections and constraints. In Figure 9, we have plotted the optimum efficiencies we found along with those of some of the coherent PSK and PAM selections presented above.

8. DISCUSSION

We can summarize this investigation by observing that more and more levels of PAM can improve the efficiency-bandwidth tradeoff in a photon-counted system, but with quite diminishing returns. We see that one can achieve about 1.7 bits (1/0.6) per signaling slot at 6 dB photons per source bit, with small further increases in bandwidth efficiency at the efficiency cost of several dBs more. (We did not investigate higher than L=4 because the efficiency seemed relatively too poor, although a system designer interested in extreme versions of this approach might be satisfied with such a tradeoff.) This is quite different from the performance of the coherent modulations, where the first several factors of bandwidth improvement come for hardly any extra power cost when using larger alphabet sizes.

It is of interest to note that, in these photon-counted PAM systems, the channel code rates that achieve bandwidths of 1-1.5 bits per slot are smaller than for the OOK system (L=2, bandwidth between 1 and 2.) This may actually be a desirable characteristic if interleaving is needed to defeat channel fading.

If one wanted to implement one of these formats, near-capacity-achieving performance could be achieved by using an encoder/mapper similar to the Barron[16] approach where the several signal levels would need to be explicitly included in the constellation shaping. One could also stick with a simpler, non-capacity-achieving design (such as Hisdal's[22] but one would need to assess whether the poorer efficiency-bandwidth tradeoff were still worth it.)

We have also investigated the penalty of photon-counted PAM in background noise. (See also [11].) If one knew the noise value one expected, one could re-optimize for capacity by including the noise term in the capacity formula (as presented earlier.) We have calculated performance for the optimized-for-noiseless designs in varying amounts of noise, and have found that, for those that use medium to high channel code rates, the noise penalty is several dBs poorer than the L=2 versions. We believe that this is because the L>2 PAM designs include particularly small pulse levels.

We observe that noiseless L=2 designs can use photon-counting receivers that only need to put out "none" or "one or more" count measurements. It is known, though, that in a noise background, actual photon counts give better performance than mere 0/1. Similarly, our multi-level PAM signaling can only approach these capacity values with actual photon counts (and full soft decision coding. Remember that we used the full Poisson distribution in our capacity formulation.)

Finally, we return to our original goal of seeing if a photon-counting telescope/receiver could be used for both high-efficiency/low data rate parts of a mission (ie, the max range) and the higher data rate (shorter range) parts of the mission. Photon counting receivers need to be sized to be able to handle the expected flux (count rate) with little penalty. Such a short range / long range receiver would thus need to handle the high count rates of the shorter ranges, which is likely a technology driver.

If the high short-range flux could be handled, and if the data rates could be kept below about ten times the technologically max slot rates, then the photon-counting receiver (with two-level modulation) is probably the best choice because of its excellent efficiency properties. However, if technological limits prevent the slot rates from growing as fast as the desired data rates, one might approach the 0.7<BW<3 region where some of the multi-level techniques discussed here would be applicable. The system designer would need to assess whether the performance is adequate. We note that one could keep efficiency high, instead, by adding the complexity of wavelength division multiplexing of multiple lower-rate but higher-efficiency, binary signaling, at both the transmitter and receiver. The system designer would need to assess the tradeoffs of higher system complexity versus larger telescopes for achieving higher data rates at the different ranges.

9. ACKNOWLEDGMENTS

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11. FIGURES

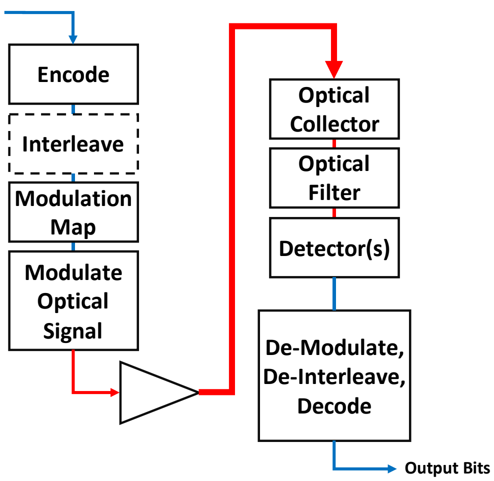


Figure 1 – Channel model

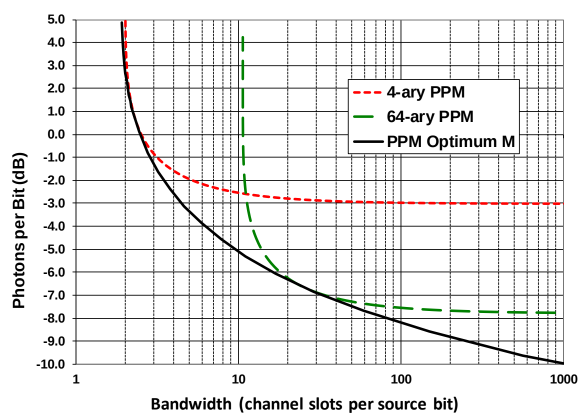
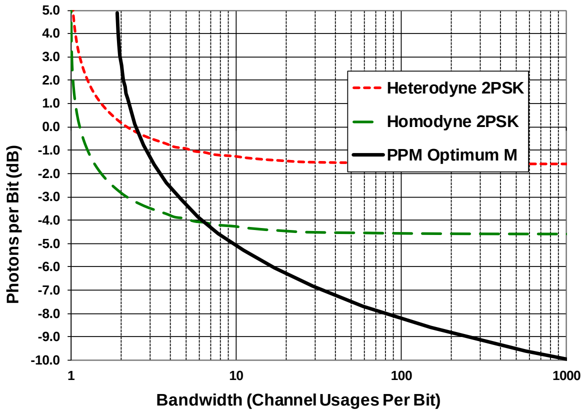


Figure 2 – Efficiency at capacity for photon-counted PPM 4 & 64 and optimized



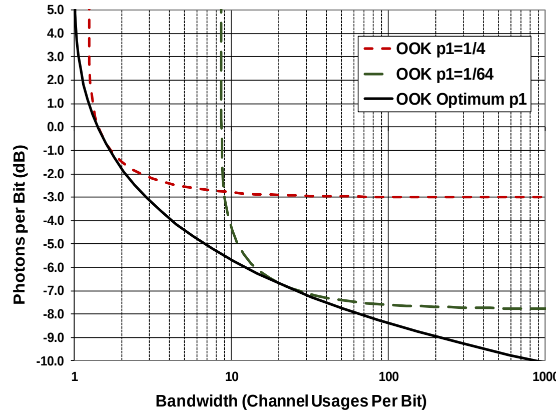
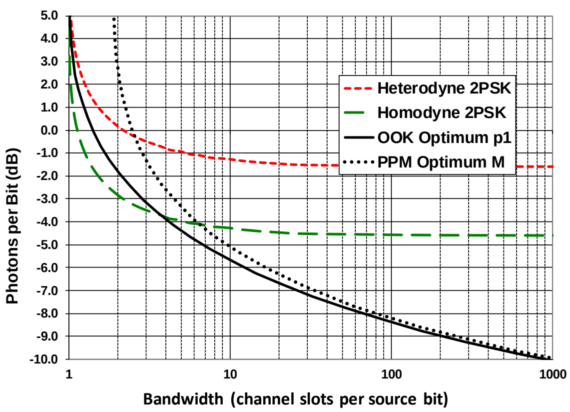


Figure 3 – Efficiency at capacity for 2PSK heterodyne and homodyne, plus optimized photon-counted PPM

Figure 4 – Efficiency at capacity for photon-counted OOK, p_1=1/4, 1/64 plus optimized



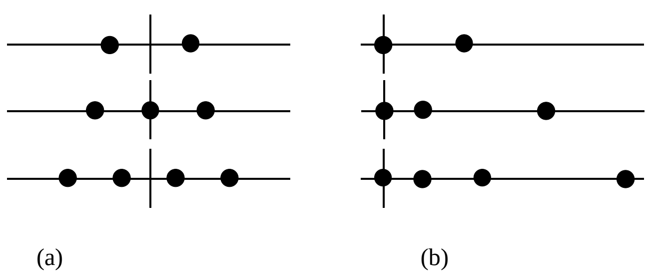


Figure 5 – Heterodyne and homodyne 2PSK, plus photon-counted optimized OOK and PPM

Figure 6 – L=2,3, & 4 PAM signaling;
(a) amplitudes for coherent, with equal probabilities,
(b) powers for photon-counting, with (to be) optimized levels and probabilities

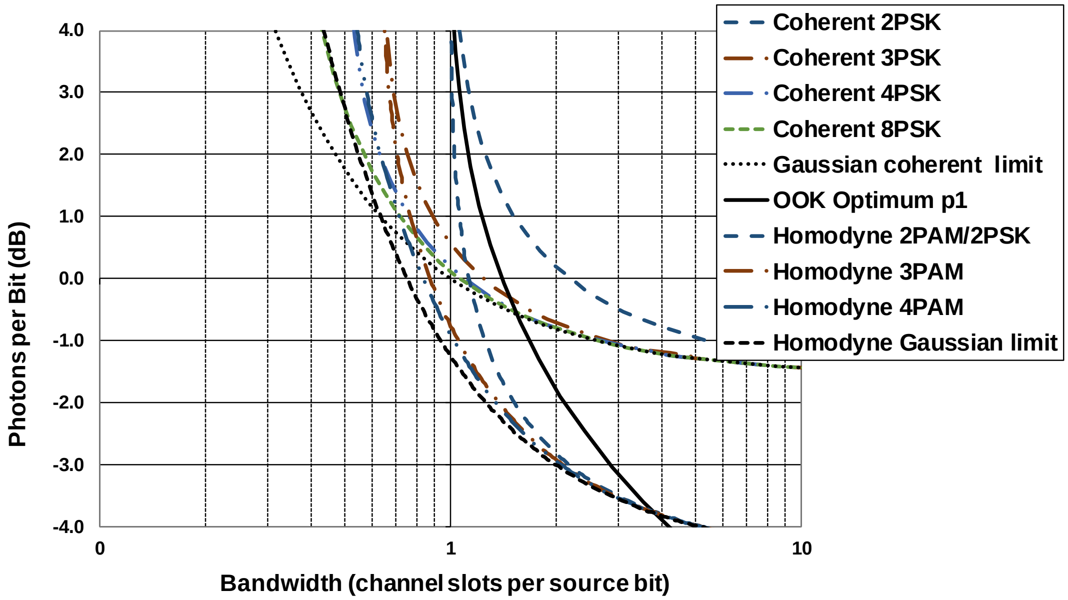
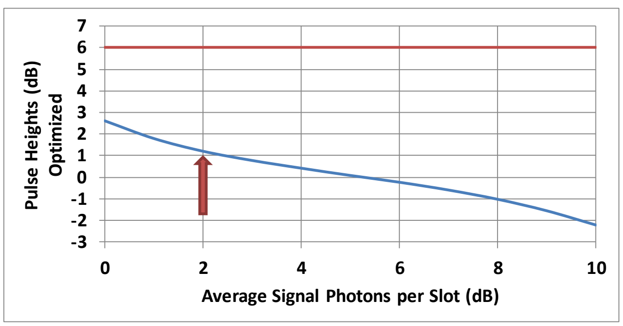
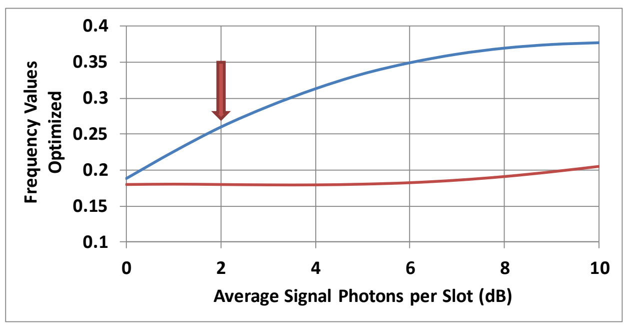


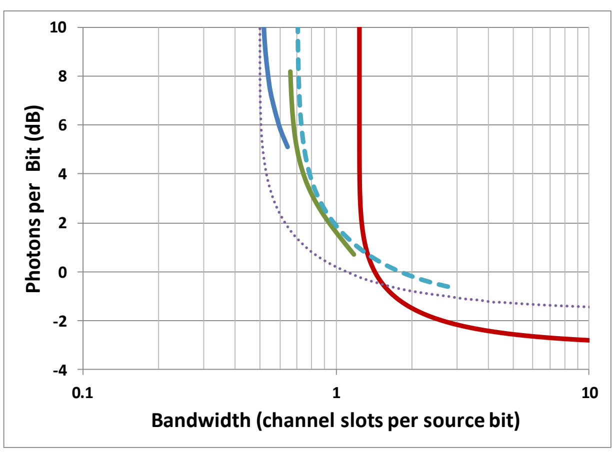
Figure 7 – Coherent (heterodyne) 2,3,4,8 PSK, homodyne 2,3,4 PAM, photon-counted optimized OOK and heterodyne and homodyne Gaussian limits



(a)



(b)



(c)

Figure 8 – Optimized, photon-counted PAM designs;
(a) L=3, M=4 signal amplitude (in dB) optimized designs;
(b) L=3, M=4 signal probability optimized designs;
(c) efficiency at capacity for optimized M=4, L=2, 3, and 4; also for fixed design (dashed, marked by arrow in (a),(b)); coherent 4PSK (dotted) shown for comparison

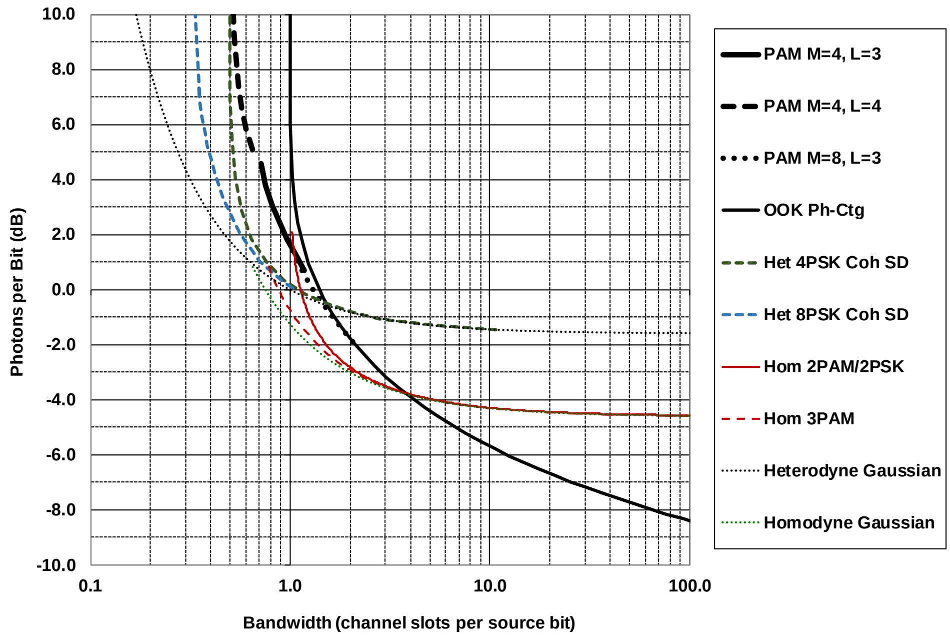


Figure 9 – Photon-counted PAM: thick black: M=4, L=3,4; M=8, L=3; also: OOK (L=2) for optimized M; for comparison – heterodyne 4&8PSK, homodyne 2&3 PAM, Gaussian optimal