A new method for analysing photon-scattering in high-speed turbulence

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ABSTRACT

Optical scattering caused by the high-speed turbulence will reduce the accuracy of optical sensors on aircraft. The existing research mainly focuses on the scattering field on the macro scale. A new method to analyse the photon-scattering characteristics in high-speed turbulence from the micro point of view is proposed in this paper, which can explain the micro disturbance of airflow molecules to photons. In order to construct the relationship between micro disturbance and macro distortion, molecular clusters are proposed to describe the multi-molecular scattering field, and the scattering characteristics of molecular clusters with wavelength, density and vortex shape are obtained by statistical method. The simulation results show that the coherent superposition of scattered waves is the dominant factor of the micro scattering characteristics of molecular clusters. The method proposed in this paper provides a new idea for the study of aero-optics under complex flow conditions.

Keywords: Optical sensors, photon-scattering, high-speed turbulence, photon-scattering, molecular clusters, micro disturbance, aero-optics

1. INTRODUCTION

The disturbance of light by the high-speed turbulence is a key problem in the study of aero-optical effects[1-5]. The essence of the process can be attributed to the scattering of photon energy in multiple directions after the "collision" between airflow molecules and photons[6]. At present, the scattering analysis of light in high-speed turbulence is to simplify the airflow molecules into a macroscopic polymer called "the transmission medium"[7-9]. For the low-speed simple flow field, its spatial pulsation is small. Because the range of local uniform distribution of airflow molecules is much larger than one wavelength, the scattered energy caused by the air flow molecules counteracts each other in all directions, as shown in Figure 1. As a result, the transmission direction of light becomes a composite direction, also known as the refraction direction of the medium[10-11].

However, the local uniform range of airflow molecules will decrease with the increase of flow complexity[12]. For hypersonic vehicles in complex flight environment, the local uniform range of the surface flow field may be reduced below the wavelength and the spatial pulsation is strong, which will make the scattered energy unable to be offset in all directions, as shown in Figure 2. Therefore, for the non-uniform flow on the surface of the hypersonic vehicle, it is necessary to analyse the generation and evolution of optical distortion with a micro theory of turbulent molecules and photon scattering.

2. MICROSCOPIC SCATTERING FIELD MODEL OF THE SINGLE MOLECULE

In order to describe the main energy transmission bodies (photons and airflow molecules) in turbulence at the micro level, a micro analysis method is established by using quantum scattering theory. A photon is described by the wave function ψ in quantum theory. Its statistical significance is the probability of finding photons at a certain position in space, and the probability value is $|\psi|^2$. When photons encounter turbulent molecules, the energy of the photons will radiate in different directions in space, as shown in Figure 3. The wave function of the radiation field can be expressed as[13]

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$$\psi(r,\theta,\varphi)=A\left[\exp(ikz)+\frac{f(\theta,\varphi)}{r}\exp(ikr)\right] \tag{1}$$

where r , θ and φ are the spherical coordinates centered on the scattering molecules. $A\exp(ikz)$ is the energy transmitted in the original direction. The energy scattered in all directions is expressed as

$$A\frac{f(\theta,\varphi)}{r}\exp(ikr)$$

where $f(\theta,\varphi)$ is the scattering amplitude of each direction in spherical coordinates.

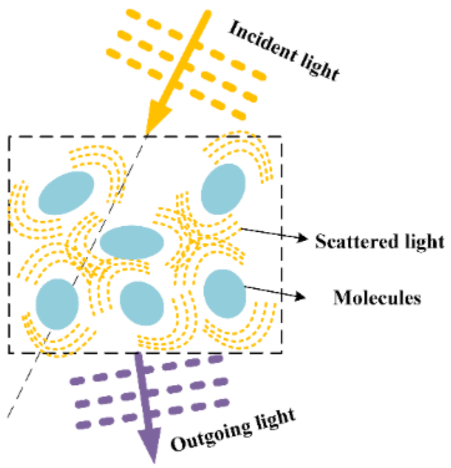

Figure 1. Scattered field caused by the nearly uniformly distributed airflow molecules.

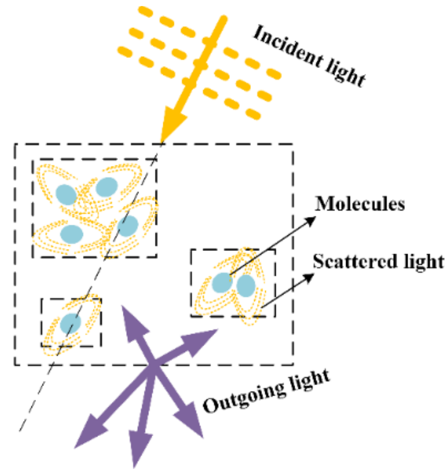

Figure 2. Scattered field caused by the non uniformly distributed airflow molecules.

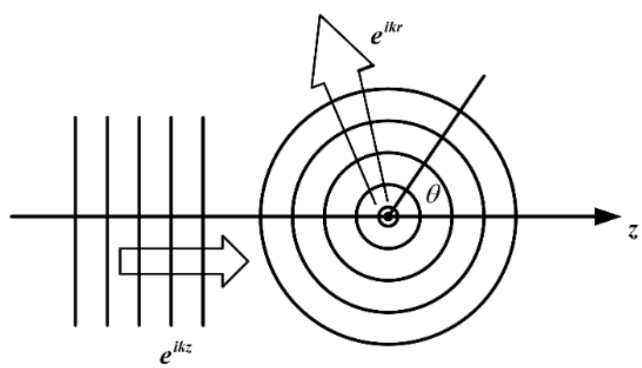

Figure 3. Schematic diagram of photon-scattering for the plane wave.

In this paper, the scattering amplitude of airflow molecules with spherical symmetrical shape can be simplified to the symmetrical form $f(\theta)$,which is determined by Schrodinger equation. The hard sphere model is used to describe the potential field of spherical molecules[14], and the scattering amplitude expression is

$$f(\theta)=\sum_{l=0}^{+\infty} i(2l+1)\frac{j_l(ka)P_l(\cos\theta)}{kh_l^{(1)}(ka)} \tag{2}$$

For the airflow molecules, the effective range a of the scattering potential field is taken as the true molecular radius, which satisfies

$$a=d_m/2, \quad d_m=3.5\times10^{-10}\text{m} \tag{3}$$

According to the equations (1), (2) and (3), it can be obtained that the wave function of the scattered field caused by the single airflow molecule is expressed as[15]

$$\psi(r,\theta)=A\left[\exp(ikz)+\frac{1}{r}\left(\sum_{l=0}^{+\infty} i(2l+1)\frac{j_l(k\,d_m/2)P_l(\cos\theta)}{kh_l^{(1)}(k\,d_m/2)}\right)\exp(ikr)\right] \tag{4}$$

The scattering amplitude of the single airflow molecule for the light of different wavelengths is obtained by equation (4), as shown in Figure 4.

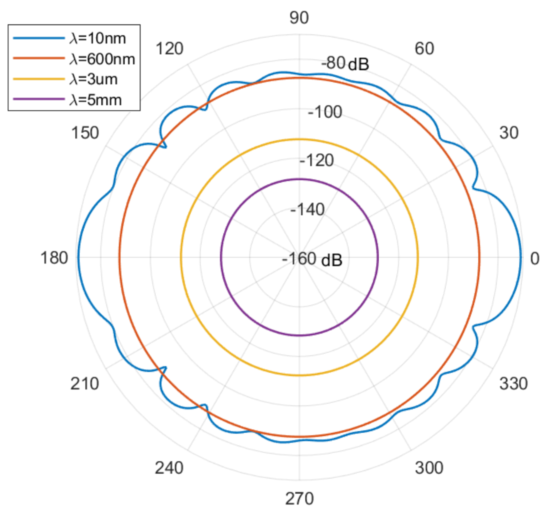

Figure 4. The scattering amplitude for the light of different wavelengths.

The integral area of the polar coordinate curve in Figure 4 represents the total scattering probability. It can be seen that the total scattering probability increases with the decrease of wavelength. Since the refraction is the synthesis of micro scattering, the change trend of the integral area corresponds to the increasing trend of the refractive index of macro medium with the decrease of wavelength.

3. MICROSCOPIC SCATTERING FIELD MODEL FOR THE MULTI MOLECULES

The high-speed flow field is a macro medium composed of a large number of airflow molecules. Based on the scattering mechanism of a single molecule, the model of the scattering field caused by the multi molecules is established. The received field is the superposition of the scattered field of each molecule. According to the scattering model of the single molecule, the wave function of a single molecule is composed of the transmitted wave and the scattered wave. The total wave function is the superposition of the wave functions of a lot of transmitted photons and scattered photons, which has a stable statistical law in the macro level. Therefore, the probability wave function in the quantum theory can be transformed into the deterministic electric field measured by macro photoelectric sensor, which is expressed as

$$\begin{aligned}\sum_{m=1}^{M_t}\psi_m(r,\theta)&\xrightarrow{\text{Statistics}}\boldsymbol{E}_{\text{total}}(r,\theta)=\boldsymbol{E}_t+\boldsymbol{E}_s(r,\theta)\\&=A\exp\left(ikd_{S\to T}\right)+A\sum_{m=1}^{M_t}\frac{f(\theta)}{r_m}\exp\left[ik\left(r_m+d_{S\to m}\right)\right]\end{aligned} \tag{5}$$

where $\boldsymbol{E}_t$ is the transmitted field. $d_{S\to T}$ represents the distance from the wave source to the receiving point. $\boldsymbol{E}_s(r,\theta)$ is the scattered field. r_m is the distance from the receiving point to the m-th scattering molecule.

It can be seen from equation (5) that the larger the total number of molecules M_t, the more the total energy scattered. However, the wave function has the phase property, which will affect the total field characteristics during statistical superposition. We found that the distribution of molecules (affecting r_m) and wave number k also affect the received field besides the total number of molecules.

4. MICROSCOPIC SCATTERING ANALYSIS BASED ON THE MOLECULAR CLUSTERS

When the complex density structure of the flow field is subdivided enough, the uniformly distributed parts can always be obtained, which contain multiple molecules, called molecular clusters. In this paper, the molecular clusters are regarded as the basic calculation unit of the photon scattering. The scattering field model for the multi molecules is used to describe the optical properties of molecular clusters. In this section, the effects of the scale, density and shape of molecular clusters and the wavelength of incident photons on scattering are analyzed.

4.1 Influence of the scale of molecular clusters and the wavelength of incident photons on scattering

We selected three common wave bands for analysis, as shown in Table 1.

Table 1. The common wave bands.

	Wavelength	Application
Case 1	600 nm	Celestial navigation
Case 2	3-5 μm	Infrared acquisition
Case 3	5 mm	Millimeter wave radar

For the small uniformly distributed molecular clusters, the shape is described by the isotropic spherical vortices. The scale Λ is expressed by the diameter of the spherical vortex model, expressed as $\Lambda = D$.

The scattering intensity of the spherical vortex molecular clusters is shown in Figure 5. It can be seen that the scattering intensity of $\lambda = 600nm$ and $\lambda = 3\mu m$ is very close under the same scale of molecular clusters, which can be explained by the superposition mechanism of multi-molecular scattering field.

Under the influence of $f(\theta)$ and $\sum_{m=1}^{M_1} \frac{f(\theta)}{r_m} \exp\left[ik\left(r_m + d_{S\to m}\right)\right]$, the scattering field of molecular clusters will have the strongest scattering in a certain wave band, as shown in Figure 6. The scattering energy of the spherical vortex model for molecular cluster is the strongest in the visible and near-infrared bands.

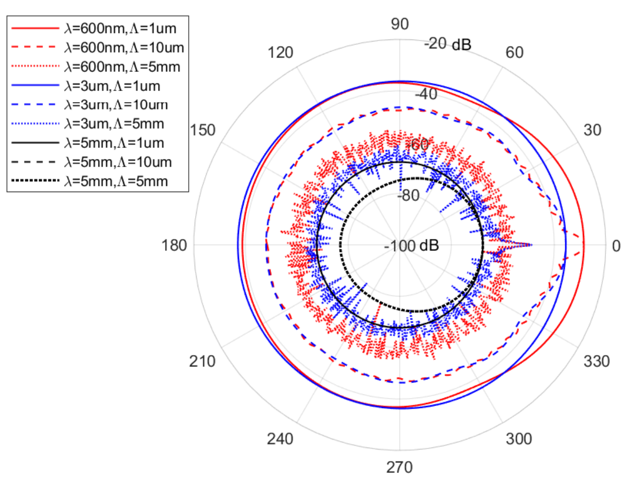

Figure 5. Scattering intensity of the spherical vortex molecular clusters.

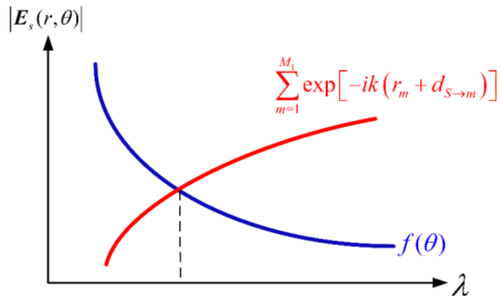

Figure 6. Variation of scattering intensity of molecular clusters.

It can also be seen from Figure 5 that the directionality of scattering intensity $\left|\boldsymbol{E}_s\left(r,\theta\right)\right|$ increases with the increase of $\frac{\Lambda}{\lambda}$. In other words, the waves with scales far larger than Λ are isotropically scattered, while the scattering directivity of the waves with scales smaller than Λ is refraction. The results show that the micro scattering theory is consistent with the traditional macro analysis method when applied to uniform large-scale media.

4.2 Influence of the shape of molecular clusters on scattering

In order to analyze the influence of the shape of molecular cluster on scattering, the different models of molecular clusters were simulated and analyzed. The vortex structure in the real turbulence is usually ellipsoidal or hairpin. Therefore, the scattering characteristics of ellipsoidal vortex and hairpin vortex are analyzed in this section.

Figures 7 and 8 show the scattering characteristics of two vortex models of molecular clusters respectively. It can be seen from the comparison with the spherical vortex molecular clusters: (A) The difference of scattering intensity $\left|\boldsymbol{E}_s\left(r,\theta\right)\right|$ is mainly reflected when $\frac{\Lambda}{\lambda}$ is large, which is characterized by macro refraction; (B) The microscopic scattering is still isotropic when $\frac{\Lambda}{\lambda} \ll 1$; Macro refraction is caused by the change of the angle between the incident and the outgoing of the molecular clusters, which can be described by the refraction law. The reason of microscopic isotropic scattering is that the energy cancellation caused by long wave coherent superposition is weak.

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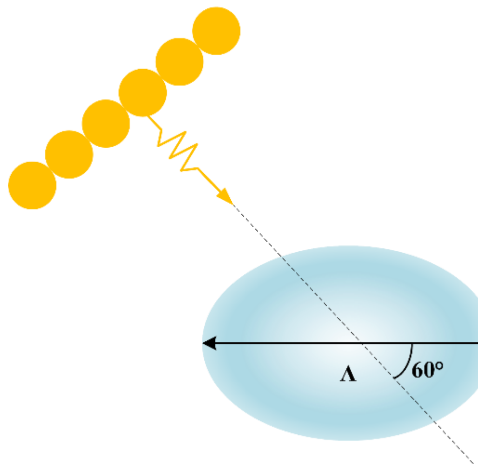
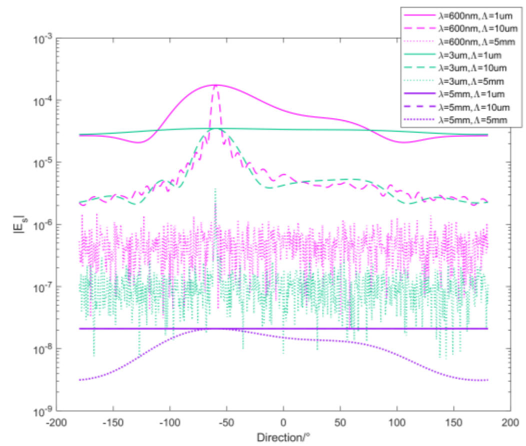
4.3 Influence of the density of molecular clusters on scattering

Since the scattering direction characteristics of molecular clusters are affected by $\frac{\Lambda}{\lambda}$, two cases for influence of density are discussed.

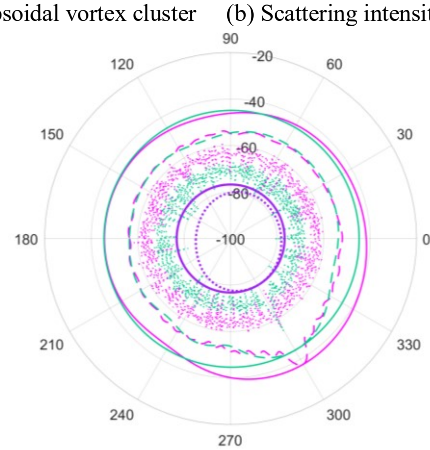
4.3.1 Obvious Directionality of Scattering for the Case of $\frac{\Lambda}{\lambda} \gg 1$. In order to avoid the influence of irregular surface, the rectangle with $\Lambda \gg \lambda$ is used as the distribution area of molecular clusters, as shown in Figure 9. The direction of anisotropic scattering is expressed by the expected direction of scattering of molecular clusters. The deviation angle of the expected direction and the incident direction is shown in Figure 10. It can be seen from Figure 10 that the deviation angle increases with the increase of molecular number, which actually reflects the positive correlation between the refractive angle of macro medium and density.



(a) Schematic diagram of ellipsoidal vortex cluster



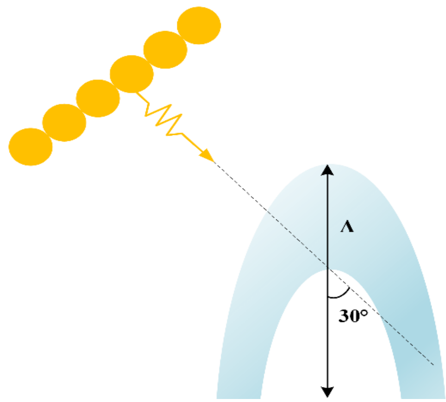
(b) Scattering intensity (rectangular coordinates)



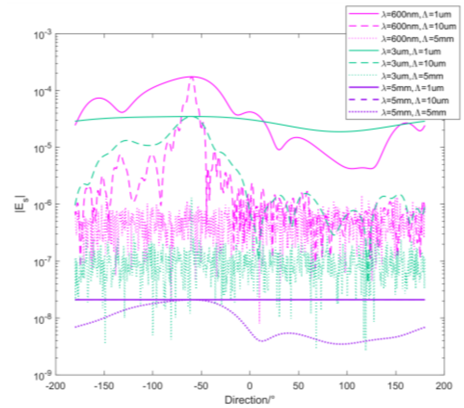
(c) Scattering intensity (polar coordinates)

Figure 7. Scattering intensity of ellipsoidal vortex cluster.

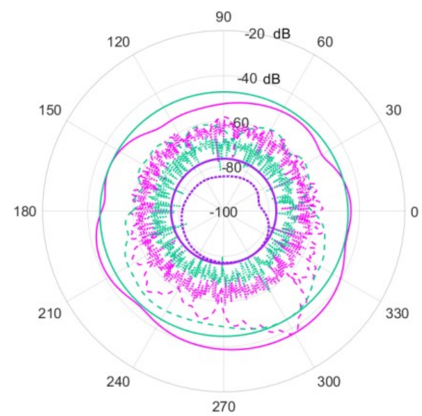
4.3.2 Isotropic Scattering for the Case of $\frac{\Lambda}{\lambda} \ll 1$. Since the scattering probability in each direction caused by molecular clusters of smaller scale is very close, the Influence of the density on scattering intensity is described by the mean square root of scattering amplitude $rms\left(\left|\boldsymbol{E}_s\left(r,\theta\right)\right|\right)$ in each direction, as shown in Figure 11. It can be seen that the molecular number and $rms\left(\left|\boldsymbol{E}_s\left(r,\theta\right)\right|\right)$ always show a linear relationship for different wavelengths. This is because the scattered spherical wave is not sensitive to the phase difference caused by the randomly distributed molecules in the small-scale molecular clusters, and the coherent superposition of the scattered waves in each direction is approximately the amplitude superposition of the scattered waves of each molecule. Therefore, the scattering intensity of the whole molecular clusters is directly proportional to the total number of molecules. When calculating the distortion caused by the density field in a macro region, the amount of calculation required to directly simulate all molecules in the molecular clusters is too large. Therefore, the linear relationship between scattering intensity and molecular number can be fitted, and then the scattering intensity of molecular clusters for different density can be calculated.



(a) Schematic diagram of hairpin vortex cluster



(b) Scattering intensity (rectangular coordinates)



(c) Scattering intensity (polar coordinates)

Figure 8. Scattering intensity of hairpin vortex cluster.

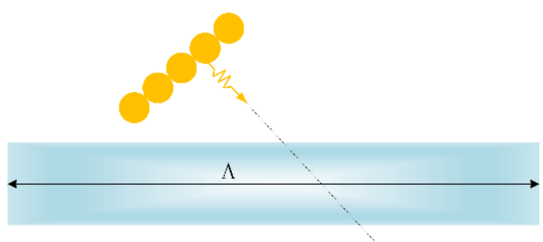


Figure 9. Schematic diagram of rectangular molecular clusters.

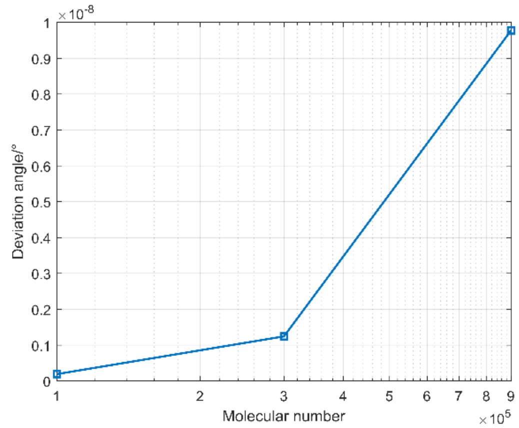


Figure 10. The deviation angle of the expected direction and the incident direction.

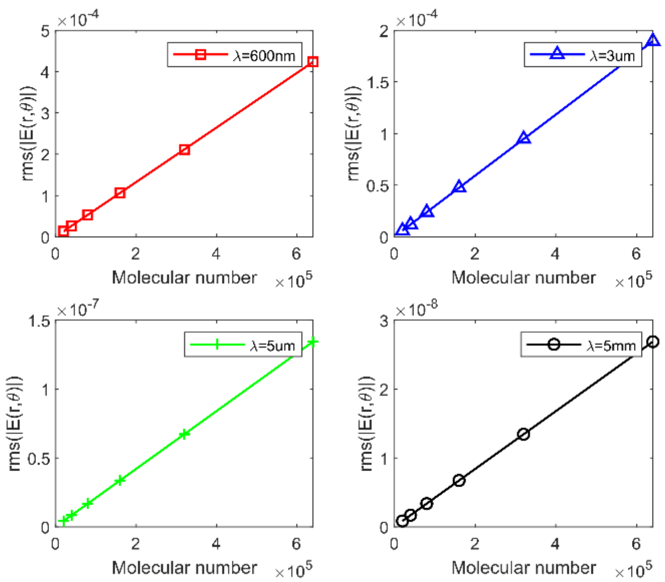


Figure 11. The influence of the molecular number on scattering intensity.

5. CONCLUSION

In order to analyze the photon scattering characteristics in high-speed turbulence, a new micro analysis method based on molecular clusters is proposed in this paper, which can explain the micro disturbance of airflow molecules to photons.

Through the statistical analysis of the optical characteristics of molecular clusters, we find that the scattering characteristics caused by large-scale molecular clusters ($\frac{\Lambda}{\lambda} \gg 1$) are the same as the refraction law in macro geometric optics. The directionality of the scattering is obvious, and it is consistent with the aero-optical effect caused by large-scale turbulent structure in the flow field. There is no obvious directionality for the scattering of small-scale molecular clusters ($\frac{\Lambda}{\lambda} \ll 1$), and the proportion of the transverse and backward energy loss relative to the forward transmission energy cannot be ignored. The macro analysis method cannot deal with the energy loss in the process of photon transmission. The micro method should be used to calculate the aero-optical characteristics for high-density and small-scale structures in high-speed flow field. Moreover, the photon scattering theory has the advantage of energy analysis, which is more suitable for the image compensation of aero optical effect under complex flow conditions.

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