Adaptive backstepping sliding mode control of manipulator based on nonlinear disturbance observer

Lirong Yang^{a*}, Jiajun Li^a, Lumin Tan^b

^a School of Mechanical and Electrical Engineering, Jiangxi University of Science and Technology, Jiangxi, China; ^b School of Electrical Engineering and Automation, Jiangxi University of Science and Technology, Jiangxi, China

ABSTRACT

To enhance the precision and efficiency of manipulation trajectory tracking, a trajectory tracking control strategy combining nonlinear disturbance observer and adaptive inverse sliding mode control is designed. Prior to designing the operator trajectory tracking control strategy, the general kinetic model of the manipulator must be constructed. For the observable part of the disturbance signal, the nonlinear disturbance observer is used for online observation, and the observation error converges exponentially by selecting appropriate parameters; For the unobservable part, the adaptive law is used to compensate, and the Lyapunov stability theory establishes the system's stability. According to the simulation results, the other two methods, the approach suggested in this paper not only speeds up the response speed of the system but also can better overcome the uncertainty of the object and external random interference, significantly reducing system chatter and enhancing the system's ability to exert control.

Keywords: Trajectory tracking, nonlinear disturbance observer, sliding mode control, adaptive law, observation error

1. INTRODUCTION

The manipulator is a highly linked, non-linear, time-invariant system, and its core is the control system. In order to achieve high-precision fast tracking and reduce the influence of uncertain factors, an appropriate control strategy must be selected. Common control methods include proportional-integral and derivative control, robust control, sliding mode control, etc. The advantage of proportional-integral and derivative control is that it does not depend on an exact mathematical model., but the starting torque is too large during control, which is easy to damage the manipulator¹. Robust control can be well used in systems with uncertain parameters, but for nonlinear systems, the upper limit of uncertainty cannot be accurately quantified². The sliding mode controll⁴⁻⁵. Many scholars have researched suppressing chattering. References⁶⁻⁷ propose to combine the boundary layer method and the sliding mode control algorithm. References⁸⁻⁹ design a sliding mode differential interference observer with a non-singular terminal adaptive power exponential approach rate, which achieves rapid system response while avoiding chattering. References¹⁰⁻¹¹ use a recursive design approach that blends inversion control and sliding mode control, which not only makes it easier to build inversion control but also makes the system more resilient. In this research, an adaptation inverse slipping mode control method which is based on an enhanced nonlinear disturbance observer is developed to improve the response time and control accuracy of the manipulator system.

2. MATHEMATICAL MODELING OF MANIPULATOR

By the Lagrange method, the space has n joint manipulator in the general dynamic equation are as shown below¹²:

$$M(q)q + C(q,q)q + G(q) = \tau + d \tag{1}$$

(1)

where $M(q) \in R^{n \times n}$ is the matrix of the positive deterministic inertia, $G(q) \in R^n$ is the vector of gravity term acting on the joint, $C(q,\dot{q}) \in R^{n \times n}$ is the centrifugal force and Coriolis Forces Matrix, $\tau \in R^n$ is the control torque, and $d \in R^n$ is the modeling uncertainty, Angular displacement (or position), angular speed and angular acceleration of each manipulator

* 906462005@qq.com

Third International Conference on Computer Science and Communication Technology (ICCSCT 2022) edited by Yingfa Lu, Changbo Cheng, Proc. of SPIE Vol. 12506, 125062T © 2022 SPIE · 0277-786X · doi: 10.1117/12.2661775 joint are indicated by the letters $q \dot{q}$ and \ddot{q} . The dynamical characteristics of the system of manipulators depicted by equation (1) are as shown below.

Characteristic 1: Positive values α and β satisfy the following inequality, and inertia matrix q is a symmetric and limited positive definite matrix.

$$\alpha x \, \| \le x^T M(q) x \le \beta x \, \| \tag{2}$$

(**a**)

Characteristic 2: $M(q) - 2C(q, \dot{q})$ is an obliquely symmetric matrix.

Characteristic 3: There exists a parametric vector that will depend on the manipulator parameters such that M(q), $C(q,\dot{q})$, G(q) satisfy a Linear relationship.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Phi(q,\dot{q},\ddot{q})P$$
(3)

where $P \in \mathbb{R}^n$ is the unknown constant parameter vector representing the mass properties of the manipulator and $\Phi(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n \times m}$ is the function matrix of the known generalized coordinates and derivatives of the manipulator.

3. CONTROLLER DESIGN

System structure diagram is shown in Figure 1. q_d is the angle command signal given by the manipulator, \dot{q}_d is the given angular velocity command signal, τ_{BS} is the virtual control rate output by the adaptive inversion controller, and τ is the obtained actual input torque, τ_D is the output control law of the interference viewer¹³.



Figure 1. Diagram of the control system's structure.

3.1 Design of nonlinear disturbance observer

The objective of the nonlinear disturbance observer are as follows:

$$\begin{aligned} \begin{vmatrix} \dot{z} &= -L(q,\dot{q})\dot{d} + L(q,\dot{q})(C(q,\dot{q})\dot{q} + G(q) - \tau) \\ \dot{d} &= z + \varphi(\dot{q}) \end{aligned}$$
(4)

where the observation error $D = d - \hat{d}$, \hat{d} is the observed value of d, $L(q,\dot{q})$ is the gain matrix, if formula (4) satisfies:

$$\varphi(\dot{q}) = \varepsilon \begin{bmatrix} \dot{q}_1 \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix}$$
(5)

Then the interference observer index converges and can approximate the actual value, where $\varepsilon > 0$.

Proof: because

$$\dot{\phi}(\dot{q}) = \varepsilon \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_1 + \ddot{q}_2 \end{bmatrix} = \varepsilon \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \ddot{q} = L(q, \dot{q}) M(q) \ddot{q}$$
(6)

Therefore

$$L(q,\dot{q}) = \varepsilon \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} M^{-1}$$
(7)

The Lyapunov function is defined as V_0 , its derivation can be obtained:

$$\dot{V}_0 = D^T \dot{D} = -D^T L D \tag{8}$$

If the minimum eigenvalue of matrix $L(q,\dot{q})$ is λ_{\min} , it can be obtained from equation (8), $\dot{V}_0 \leq -\lambda_{\min} \|D\|^2 = -2\lambda_{\min}V_0$. The solution is $V_0(t) \leq e^{-2\lambda_{\min}(t-t_0)}V_0(t_0)$. t_0 is the initial value of time t, it can be concluded that the observer can converge exponentially. The manipulator system described in equation (1) can be written as:

$$\ddot{q} = M^{-1}(\tau_{BS} + D - C\dot{q} - G)$$
(9)

It can be seen from the above formula that the total interference is reduced after using the observer.

3.2 Design of backstepping sliding mode controller

It is possible to rewrite the manipulator system with disturbance observer as follows:

$$\begin{cases} \dot{x}_1 = x_2 = \dot{q} \\ \dot{x}_2 = \ddot{q} = M^{-1}(\tau_{BS} + D - C\dot{q} - G) \end{cases}$$
(10)

Let q_d be the position command signal given by the manipulator, and let q be the actual output. The precise processes for designing the controller are as follows:

Step 1: the tracking error is defined as $z_1 = q - q_d$, the derivation leads to: $\dot{z}_1 = \dot{q} - \dot{q}_d = x_2 - \dot{q}_d$. Take virtual control quantity $\alpha_1 = c_1 z_1$, where $c_1 \in R^{n \times n}$ is a symmetric positive definite constant matrix. Let $z_2 = \dot{z}_1 + \alpha_1 = \dot{q} - \dot{q}_d + \alpha_1$, the derivation leads to:

$$\dot{z}_{2} = \ddot{q} - \ddot{q}_{d} + \dot{\alpha}_{1} = M^{-1}(\tau_{BS} + D - C\dot{q} - G) - \ddot{q}_{d} + \dot{\alpha}_{1}$$
(11)

The Lyapunov function is defined as V_1 . The derivation leads to:

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T (z_2 - c_1 z_1) = z_1^T z_2 - z_1^T c_1 z_1$$
(12)

If $z_2=0$, then $\dot{V}_1 \le 0$, so the next step of design is required.

Step 2: the Lyapunov function is defined as $V_2 = V_1 + \frac{1}{2}s^Ts$. Where *S* is the sliding surface function, which is defined as: $S = k_1 z_1 + z_2$, where $k_1 \in \mathbb{R}^{n \times n}$ is a symmetric, normally-invariant matrix. The derivation leads to: $\dot{s} = k_1 \dot{z}_1 + \dot{z}_2 = k_1 (z_2 - c_1 z_1) + M^{-1} (\tau_{BS} + D - C\dot{q} - G) - \ddot{q}_d + c_1 \dot{z}_1$.

Take the derivative of V_2 :

$$\dot{V}_{2} = \dot{V}_{1} + s^{T} \dot{s} = z_{1}^{T} z_{2} - z_{1}^{T} c_{1} z_{1} + s^{T} [k_{1} (z_{2} - c_{1} z_{1}) + M^{-1} (\tau_{BS} + D - C\dot{q} - G) - \ddot{q}_{d} + c_{1} \dot{z}_{1}]$$
(13)

3.3 Design of adaptive law

The Lyapunov function is defined as $V_3 = V_2 + \frac{1}{2\eta} \tilde{D}^T \tilde{D}$, and η is a normal number. By deriving it, we can get:

$$\dot{V}_{3} = \dot{V}_{2} + \frac{1}{\eta} \tilde{D}^{T} \dot{\tilde{D}} = \dot{V}_{2} - \frac{1}{\eta} \tilde{D}^{T} \dot{\tilde{D}} = z_{1}^{T} z_{2} - z_{1}^{T} c_{1} z_{1} + s^{T} [k_{1} (z_{2} - c_{1} z_{1}) + M^{-1} (\tau_{BS} + \hat{D} - C\dot{q} - G) - \ddot{q}_{d} + c_{1} \dot{z}_{1}] - \frac{1}{\eta} \tilde{D}^{T} [\dot{\tilde{D}} - \eta (M^{-1})^{T} s]$$
(14)

The control rate designed is:

$$\begin{cases} \tau_e = M[-k_1(z_2 - c_1z_1) + M^{-1}(C\dot{q} + G - \hat{D}) + \ddot{q}_d - c_1\dot{z}_1 - \gamma s], \\ \tau_s = -\delta M\gamma \operatorname{sgn}(s), \\ \tau_{BS} = \tau_e + \tau_s. \end{cases}$$
(15)

To further increase the convergence speed of the system, the saturation function sat(s) is used instead of the symbolic

function sgn(s), as follows:

$$sat(s) = \begin{cases} sgn(s), & |s| \ge \rho, \\ \frac{s}{\rho}, & |s| < \rho. \end{cases}$$
(16)

where $\rho > 0$ is the boundary layer width on the sliding surface. Therefore, based on the improved control law, the design is

$$\tau_{BS} = M[-k_1(z_2 - c_1z_1) + M^{-1}(C\dot{q} + G - \hat{D}) + \ddot{q}_d - c_1\dot{z}_1 - \gamma s] - \delta M\gamma sat(s)$$
(17)

The adaptive law is taken as: $\dot{\hat{D}} = \eta (M^{-1})^T s$.

3.4 Stability analysis

For the whole system, the Lyapunov function V_4 is defined, and its derivation can be obtained:

$$\dot{V}_{4} = z_{1}^{T} \dot{z}_{1} + s^{T} \dot{s} - \frac{1}{\eta} \tilde{D}^{T} \dot{\tilde{D}} + D^{T} \dot{D} = z_{1}^{T} (z_{2} - c_{1} z_{1}) + s^{T} (k_{1} \dot{z}_{1} + \dot{z}_{2}) - \frac{1}{\eta} \tilde{D}^{T} (\dot{D} - \dot{\tilde{D}}) + D^{T} (\dot{d} - \dot{\tilde{d}})$$
(18)

By substituting the control law and the adaptive law into the above formula:

$$\dot{V}_{4} = z_{1}^{T} z_{2} - z_{1}^{T} c_{1} z_{1} - s^{T} \gamma s - \delta \gamma |s| - L|D|$$
(19)

Let:

$$Q = \begin{bmatrix} c_1 + \gamma k_1^2 & \gamma k_1 - \frac{1}{2} \\ \gamma k_1 - \frac{1}{2} & \gamma \end{bmatrix}$$
(20)

Because:

$$\begin{bmatrix} z_1 & z_2 \end{bmatrix} \begin{bmatrix} c_1 + \gamma k_1^2 & \gamma k_1 - \frac{1}{2} \\ \gamma k_1 - \frac{1}{2} & \gamma \end{bmatrix} \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T = z_1^T c_1 z_1 - z_1^T z_2 + s^T \gamma s$$
(21)

By selecting the appropriate values of c_1, k_1, γ , we can ensure that Q is a positive definite matrix, that is |Q| > 0, then $\dot{V}_4 \le 0$. The system can meet the theoretical conditions of Lyapunov stability, and z_1 and z_2 are gradually stable in the form of exponent, to secure the system's exponential asymptotic stability in a global sense. To ensure that each joint of the manipulator can move according to the desired trajectory.

4. SIMULATION EXAMPLE

In purpose of testing the effectiveness of the control strategy suggested in this paper, a double-joint manipulator is used as a research object and simulated by Matlab/Simulink. The parameters of the manipulator dynamic model are as follows:

$$M(q) = \begin{bmatrix} m_1 l_1^2 + m_2 (l_1^2 + l_2^2) + 2m_2 l_1 l_2 \cos(q_2) & m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) \\ m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) & m_2 l_2^2 \end{bmatrix} G(q) = \begin{bmatrix} (m_1 + m_2) g l_1 \cos(q_1) + m_2 g l_2 \cos(q_1 + q_2) \\ m_2 g l_2 \cos(q_1 + q_2) \end{bmatrix} C(q, \dot{q}) = \begin{bmatrix} -m_2 l_1 l_2 \dot{q}_2 \sin(q_2) & -m_2 l_1 l_2 (\dot{q}_1 + \dot{q}_2) \sin(q_2) \\ m_2 l_1 l_2 \dot{q}_1 \sin(q_2) & 0 \end{bmatrix}$$

The parameters of the two-joint manipulator are as follows: the masses of the connecting rod are $m_1=m_2=1.0kg$, the length of the connecting rod are $l_1=l_2=1.0m$, g=9.8, the ideal trajectories of given joint 1 and joint 2 are

 $q_{d1} = q_{d2} = 0.2 \sin(0.35\pi t)$ and $\dot{q}_{d1} = \dot{q}_{d2} = 0.07\pi \cos(0.35\pi t)$, the observable part of the interference takes the friction model as $\hat{D} = k\dot{q}$, where $k=diag(1.5 \ 1.5)$, the gain matrix is $\Lambda=diag(80 \ 80)$, the thickness ρ of the boundary layer is taken as 0.8. The initial state of the system is $[0.1 \ 0 \ 0.1 \ 0]$, the system is simulated using traditional sliding mode control, adaptive inverse slip mode control and the improved adaptive inverse slip mode controller in this paper to verify the improvement of control performance. The simulation time is set to 10s. The result of the simulation is as shown in Figures 2-4.

As Figure 2 shows that under the same initial conditions and external interference, using the traditional sliding mode control algorithm, the angle adjustment times of manipulator joints 1 and 2 are 4.8s and 4.9s respectively. Adaptive inverse sliding mode control algorithm is applied, the angle adjustment time of manipulator joints 1 and 2 are 2.9s and 2.5s respectively. Taking the improved control method of this paper, the angle adjustment time of manipulator joints 1 and 2 is 0.8s and 0.9s respectively. Similarly, the angular velocity tracking and adjustment time of manipulator joints 1 and 2 under the traditional sliding mode control algorithm are 3.9s and 4.0s respectively, the angular velocity tracking and adjustment time of manipulator joints 1 and 2 under the adaptive inversion sliding mode control algorithm are 1.8s and 1.7s respectively, and the angular velocity tracking and adjustment time of manipulator joints 1 and 2 under the improved control algorithm are 0.8s respectively.

As Figure 3 shows, the control input states of the two joints under the three algorithms reach the sliding mode surface, due to the discontinuity of control items such as the switching function, it is difficult for the system to slide strictly along the sliding mode towards the equilibrium point, so there will be serious chattering in the control input curve. When traditional sliding mode control algorithms are used. the chattering amplitudes of the control inputs of joints 1 and 2 are ± 0.15 and ± 0.14 respectively. Under the adaptive inversion sliding mode control algorithm, the buffeting amplitudes of joint 1 and 2 control inputs are ± 0.09 and ± 0.07 respectively. After the disturbance observer is added to compensate, the control input buffeting amplitudes of joints 1 and 2 are ± 0.02 and ± 0.01 separately. Figure 4 shows the observation results of the disturbance observer. It is shown from the figure that the nonlinear disturbance observer designed in this article can observe the external disturbance well, improve the system's anti-interference capability to a certain extent, and has good robustness.



Figure 2. Algorithm comparison simulation curve.



Figure 3. Control input of two joints under three algorithms.



Figure 4. Observation results of disturbance observer.

5. CONCLUSION

Aiming at the problem that the stability of the multi-joint manipulator system is affected by the uncertainty of its parameters and external disturbances, an improved sliding mode control strategy is proposed to track the motion trajectory of the manipulator. The kinetic equations of the robotic arm are first derived by applying the Lagrangian equations. Combining sliding mode inverse adaptive control methods with nonlinear disturbance observers. In this control process, a non-linear disturbance viewer is used to view the uncertainty and disturbance in the system, and then an adaptive inverse sliding mode controller is used to compensate for the unobserved part to secure the stability of the system and effectively reduce the jitter of the control input. Through computer simulation, this control method can be concluded to have the better control performance than the traditional sliding mode control and single adaptive inverse sliding mode control. It can not only observe the disturbance of the system, but also improve the response speed for the system and greatly weaken the jitter of system.

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