# Research on "time-on-target" firing method of fixed loading ammunition for naval gun 

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#### Abstract

In order to improve the firing efficiency of large-caliber naval gun, a new method of "Time-on-target (TOT)" for fixed-loading ammunition of large-caliber naval gun is proposed in this paper, and a model of TOT for fixed-loading ammunition of largecaliber naval gun is established, which verifies the feasibility of multiple simultaneous firing. Based on the naval gun attacking small and fast targets on the sea surface, combined with the naval gun tactical application, the TOT firing method of fixed ammunition of naval gun based on ladder firing is given, and the effectiveness of the method is verified by simulation.


Keywords: Naval gun, fixed-loading ammunition, error analysis, time-on-target, firing method

## 1. INTRODUCTION

In the recent maritime security situation, warships are threatened by suicide boats of good mobility and small size apart from anti-ship missiles. Currently, these suicide boats have become the biggest threat to warships. For instance, USS Cole, a destroyer of the U.S. Navy, arrived at the Gulf of Aden in October 2000, and was attacked by a fast-moving unidentified boat loaded with high explosives. In the resupply operation, the warship was damaged with a big hole by the sneaking boat, causing the death of dozens of American soldiers. In February 2017, a frigate of the Saudi Navy, Al Madinah, was hit by a suicide boat with two people killed and three injured apart from its severely damaged stern. In Yemen, Houthi rebels cracked the American blockage by sending suicide boats to slam into their planned target oil tanker, which exploded because of leaking oil pipes. Meanwhile, unmanned platforms are developed and make it very likely for numerous unmanned surface vehicles capable of attacking to approach warships for the purpose of reconnaissance or direct strike ${ }^{1}$. These targets feature small size and fast speed $^{2}$, but pose a severe threat to warships.

However, warships have limited weapons to attack small and fast targets at sea. So far, they mainly rely on large and medium caliber naval guns and small caliber naval guns for this purpose, regardless of poor effect in practice ${ }^{3}$. Maritime powers including Russia and the United States have also experienced some dilemmas in handling small and fast targets on the sea. For instance, a Russian warship had a drill with one 100 mm naval gun and one six-barrel 30 mm naval gun to shoot a boat captured from pirates. Using the 100 mm naval gun at the medium distance, it missed the target after several rounds of firing. Using the six-barrel 30 mm small caliber naval gun at the short distance, the warship also missed the target in the first two shootings with shells dropping next to or over the target. In the third and fourth rounds, the motionless boat was barely hit. The same embarrassment also happened to the American navy in a maneuver. Facing a constantly maneuvered suicide boat in the simulation, the American navy's "Phalanx" fired more than four times. In a nearly two-minute sweeping, "Phalanx" did not even hit the boat once with its shells. The American warship must have been damaged if it were a real confrontation. Striking small and fast targets on the sea has got much attention from navies around the world.
Multiple ammunition "time-on-target" (TOT) was mainly applied in the army's shelling ${ }^{4,5}$, but later introduced into the firing of a naval gun's one-dimensional trajectory correction projectile at land targets ${ }^{6-9}$. With the emergency of guided projectile, it was further applied in the shooting of guided projectiles by the naval guns ${ }^{10}$. This method imposes heightened damages onto a target with multiple shells firing at it simultaneously when the target fails to shun the shelling immediately. In other words, a
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gun can achieve the same effect of shelling as several guns during the artillery's fire support or raid. A single gun's multiple round TOT normally requires changed types of projectile or charge, or information ammunition capable of ballistic correction. It fires as many projectiles as possible in a specified order and from the preset angles, so that they can arrive at the same target area at the same time. In this way, the gun can give play to its best effect of shelling. And, the targets are always on land or onshore objects which are fixed.

Modern surface vehicles are equipped with few naval guns because of their functional design, resulting in their limited efficacy in striking small and fast targets on the sea. In this paper, an attempt is made to apply TOT in large and medium caliber naval guns striking small and fast targets on the sea, and its feasibility and effectiveness are also verified in simulation.

## 2. CURRENT STATUS OF GUNS STRIKING SMALL AND FAST TARGETS ON THE SEA

Medium and large naval guns normally employ the method of "solving meeting" in their strike at targets on the sea. Their fire effectiveness is generally evaluated on the basis of error analysis. A naval gun's single shot hit probability is calculated using the equivalent hit area of the attacked target at a certain distance, and then used to calculate its cumulative damage capability ${ }^{7}$.

Under unchanged conditions including performance and firing rate of naval gun and shooting environment, a naval gun's hit probability against small and fast targets on the sea is mainly related to the equivalent hit area of the target. The efficacy of large and medium naval guns in attacking destroyers, and small and fast targets on the sea are compared to analyze the current status of their strike at small and fast targets on the sea.
It is assumed that a frigate is 151 m long, 17.4 m wide and 9 m tall, and has the standard displacement of 4500 t . Meanwhile, a 76 mm naval gun has the firing distance of 11 km and the firing range of $90^{\circ}$. From the firing table, we can find the fall angle, distance/ orientation dispersion error of the shells fired by the naval gun based on the firing range. The equipment manual can be used to further check the naval gun's servo system error, radar error, and fire control system error, which are used to calculate its comprehensive probability error. Based on the target distance hit range and directional projection width, we can further calculate and determine that the naval gun's single shot hit probability against the frigate is 0.11 .
It is assumed that a small and fast target on the sea is an unmanned surface vehicle (USV) with 7 m length, 3 m width, 1.5 m height, and displacement 5.5 t . Similarly, it is calculated and determined that the naval gun's single shot hit probability against the USV is 0.0017 .

After further calculation, it is found that the probability of one hit out of 32 shots fired by the 76 mm naval gun is only 0.535 . For more vivid indication of the 76 mm naval gun's efficacy in shooting the small target at different distances, this paper presents the probability of one hit out of 32 shots constantly fired by the naval gun at the distance from 3 km to 12 km in the simulation.


Figure 1. Comparison of probability of firing on USV at different distances by a naval gun.

As shown in Figure 1, when a large and medium naval gun employs the traditional firing method to strike a USV or other small and fast targets on the sea, its single shot hit probability is very low, resulting in the low probability of one hit out of 32 continuous shots. Moreover, the probability decreases dramatically with the increase of firing range. The small and fast targets on the sea have poor resistance to damage (that is, destroyed upon hit), but when the firing range is 3 km , the damage probability of 32 continuous shots on a USV by a 76 mm caliber naval gun is 0.64 . When the firing range is 7 km , the damage probability of 32 shots drops to less than 0.2 . When the firing range is 11 km , the probability is only 0.05 .

If a large and medium caliber naval gun employs the traditional firing method to strike a USV or other small and fast targets on the sea, its single shot hit probability will be therefore very low because of small size and high maneuverability of the target ${ }^{11}$. The naval gun may not even hit the small and fast target on the sea while which is approaching the warship. A USV or armed speedboat which is equipped with a small anti-ship missile with the firing range of $3-4 \mathrm{~km}$ or other weapons, will become a severe threat to the surface ship. For this reason, a new firing method must be urgently developed to improve the strike of large and medium caliber naval guns against small and fast targets on the sea.

## 3. TOT FIRING MODEL FOR LARGE AND MEDIUM CALIBER NAVAL GUNS

Surface vehicles are equipped with fixed loading shells. The shells of the same type are charged in the same way, and have identical muzzle velocity and external ballistic characteristics with the same firing range. Therefore, they cannot achieve time-on-target (TOT) by altering charge ${ }^{12}$. Nevertheless, it is necessary to analyze whether time-on-target can be realized with multiple ammunition at the target within a firing range through high and low ballistics.

In order to verify the feasibility of multiple round TOT of the fixed loading shells by large and medium caliber naval guns, The simulation analysis is performed with created mass point external ballistic differential equations.
According to howitzer external ballistics, the mass point ballistic equations for projectiles are written with the independent variable $t$ in the geographical rectangular coordinate system under the basic ballistic assumptions ${ }^{8}$.

$$
\left\{\begin{array}{l}
\frac{d v}{d t}=-C H_{\tau}(y) \quad F\left(v_{r} \tau\right)-g \sin \theta  \tag{1}\\
\frac{d \theta}{d t}=-g \cos \theta / v \\
\frac{d x}{d t}=v \cos \theta \\
\frac{d y}{d t}=v \sin \theta
\end{array}\right.
$$

where the initial conditions are $v=v_{0}, \theta=\theta_{0}, x=y=0 ; C$ denotes the ballistic coefficient; $H_{\tau}(y)$ represents the air density function; and $F\left(v_{r} \tau\right)$ is the air drag function. The mass point ballistic differential equations are simplified, but still reflect the basic law and characteristics of projectile motion in the air under standard conditions. They are therefore applicable in the analysis of "time-on-target" by large and medium caliber naval guns.

Subsequently, a 76 mm naval gun with high explosive projectiles was taken as an example for the simulative analysis of external ballistics under standard atmospheric conditions. Its ballistics at different firing ranges was obtained as shown in Figures 2 and 3.
Figure 2 presents the 76 mm naval gun's simulation result of high and low ballistics at the firing range of 9.5 km . It is revealed that the naval gun can achieve TOT in high and low ballistics with its shells at such a range. Moreover, there are only high and low external ballistics with TOT at such a range since it fires fixed loading shells. The simulation result of high and low ballistics with TOT at the whole firing range of the 76 mm naval gun is illustrated in Figure 3. As revealed in the simulation, the 76 mm naval gun can theoretically achieve the high and low ballistics with TOT at the whole range.


Figure 2. The simulation result diagram of high and low ballistics with shooting method of TOT at the range of 9.5 km .


Figure 3. The simulation result diagram of high and low ballistics with shooting method of TOT at the whole range.

## 4. ANALYSIS OF TOT ERROR AND FIRING EFFECTIVENESS

Generally, a naval gun fires a moving target on the sea in the following process: the naval gun surveys the current coordinates of the target through its observation equipment, and outputs the coordinates to the fire command and control instrument; the fire command and control instrument carry out the filter, calculates the motion parameters of the target, solves the meeting, and obtains the firing data after considering a variety of correction factors; the firing data is transmitted to the naval gun servo system, so that the gun tube is turned to a specific firing direction; the naval gun fires a projectile that flies to the predicted point where the target will move to. This firing process faces errors at every step, so that it must be reasonably planned. The errors mainly include observation equipment surveying error, command and control instrument output error, naval gun servo system transmission error, and projectile dispersion error. At the predicted point of the target, these errors may correspondingly cause the projectile to deviate from the target, that is, firing error. In terms of repeatability, these errors are classified into unrepeatable errors, single gun repeatable errors and repeatable errors.

From the perspective of error analysis, time-on-target (TOT) actually converts some unrepeatable errors into salvo repeatable errors. For instance, the observation error and fire control motion parameter error of the same target belong to unrepeatable errors in the traditional approach of "solving meeting", and are calculated for each shot. However, they are transformed into
repeatable errors in the TOT method. Considering the firing error of a naval gun, the efficacy of two shots with the TOT method is subsequently analyzed.
Assuming that the naval gun fires at a typical target, and its target hit area is $21_{\mathrm{x}} \times 21_{\mathrm{z}}=20 \times 40=800 \mathrm{~m}^{2}$. The firing errors are $E_{x 1}=38.73(\mathrm{~m}), E_{x 2}=70(\mathrm{~m}), E_{x 3}=100(\mathrm{~m}) ; E_{21}=9.95(\mathrm{~m}), E_{22}=15(\mathrm{~m}), E_{z 3}=20(\mathrm{~m})$. The system error is 0 . The naval gun launches 60 shots. The probability of one hit out of 60 shots with two shooting methods is analyzed on this basis.
When the traditional method of solving meeting is employed, $E_{\mathrm{x} 1}, E_{\mathrm{z} 1}, E_{\mathrm{x} 2}$, and $E_{\mathrm{z} 2}$ are put into the first group of errors, $E_{x 3}$ and $E_{23}$ belong to the third group of errors. There are:

$$
\begin{align*}
& E_{X 1}^{\prime}=\sqrt{E_{\mathrm{x} 1}^{2}+E_{\mathrm{x} 2}^{2}}  \tag{2}\\
& E_{\mathrm{Z1}}^{\prime}=\sqrt{E_{\mathrm{z1}}^{2}+E_{\mathrm{z2}}^{2}} \tag{3}
\end{align*}
$$

Through the simulation with such two groups, it is calculated that the probability of one hit is 0.4770 .
When the TOT method is employed, two shots in a pair, that is, 30 pairs are launched at 30 predicted points. The calculation is conducted with three groups of errors.
It is assumed that $\left(\boldsymbol{x}_{2}, \boldsymbol{x}_{3}, \boldsymbol{z}_{2}, \boldsymbol{z}_{3}\right)$ is a group of specific values, so that the conditional probability of one hit is

$$
\begin{equation*}
P\left(\mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{z}_{2}, \mathrm{z}_{3}\right)=P\left(\mathrm{x}_{2}, \mathrm{x}_{3}\right) P\left(\mathrm{x}_{2}, \mathrm{x}_{3}\right) \tag{4}
\end{equation*}
$$

where

$$
\begin{gathered}
P\left(\mathrm{x}_{2}, \mathrm{x}_{3}\right)=\int_{-\mathrm{x}}^{\mathrm{x}} \frac{\rho}{\sqrt{\pi} E_{\mathrm{x} 1}} \cdot \exp \left[-\rho^{2}\left(\frac{x-\mathrm{x}_{2}-x_{3}-m_{x}}{E_{X 1}}\right)^{2}\right] d x, \\
P\left(\mathrm{z}_{2}, \mathrm{z}_{3}\right)=\int_{-\mathrm{z}}^{\mathrm{z} \mathrm{z}} \frac{\rho}{\sqrt{\pi} E_{\mathrm{zl}}} \cdot \exp \left[-\rho^{2}\left(\frac{\mathrm{z}-\mathrm{z}_{2}-\mathrm{z}_{3}-m_{\mathrm{z}}}{E_{\mathrm{z} 1}}\right)^{2}\right] d \mathrm{z},
\end{gathered}
$$

where $P\left(\mathrm{x}_{2}, \mathrm{x}_{3}\right)$ and $P\left(\mathrm{z}_{2}, \mathrm{z}_{3}\right)$ are the probability of one hit for a specific value of $\mathrm{x}_{2}, \mathrm{x}_{3}$ and $\mathrm{z}_{2}, \mathrm{z}_{3}$, respectively.
Hence, the conditional probability of one hit out of two shots in a pair with the TOT method is

$$
P_{L 1}^{1}\left(\mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{z}_{2}, \mathrm{z}_{3}\right)=1-\left[1-P\left(\mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{z}_{2}, \mathrm{z}_{3}\right)\right]^{2}
$$

Considering all the possible values of $\mathrm{X}_{2}$ and $\mathrm{Z}_{2}$, the probability of one hit in a pair is:

$$
P_{L 1}^{1}\left(\mathrm{x}_{3}, z_{3}\right)=\iint_{\infty} \phi\left(x_{2}, z_{2}\right) \cdot P_{L 1}^{1}\left(x_{2}, x_{3}, \mathrm{z}_{2}, z_{3}\right) d x_{2} d z_{2}
$$

where $\phi\left(x_{2}, z_{2}\right)=\phi\left(x_{2}\right) \cdot \phi\left(z_{2}\right)$ is the error distribution density function of the second group.

$$
\begin{gathered}
\phi\left(x_{2}\right)=\frac{\rho}{\sqrt{\pi} E_{X 2}} \exp \left[-\rho^{2} \frac{x_{2}^{2}}{E_{X 2}^{2}}\right] \\
\phi\left(\mathrm{z}_{2}\right)=\frac{\rho}{\sqrt{\pi} E_{\mathrm{z2}}} \exp \left[-\rho^{2} \frac{\mathrm{z}_{2}^{2}}{E_{\mathrm{z2}}^{2}}\right]
\end{gathered}
$$

In this way, the conditional probability of one hit in $t$ pairs is:

$$
P_{L 1}^{\mathrm{t}}\left(\mathrm{x}_{3}, \mathrm{z}_{3}\right)=1-\left[1-P_{l 1}^{\mathrm{l}}\left(\mathrm{x}_{3}, \mathrm{z}_{3}\right)\right]^{t}
$$

Considering all the possible values of $\mathrm{X}_{3}$ and $\mathrm{Z}_{3}$, the probability of one hit in $t$ pairs is

$$
P_{L 1}=\iint_{\infty} \phi\left(x_{3}, z_{3}\right) \cdot P_{L 1}^{\mathrm{t}}\left(x_{3}, z_{3}\right) d x_{3} d z_{3}
$$

where $\phi\left(x_{3}, z_{3}\right)=\phi\left(x_{3}\right) \cdot \phi\left(z_{3}\right)$ is the error distribution density function of the second group.

$$
\begin{aligned}
& \phi\left(x_{3}\right)=\frac{\rho}{\sqrt{\pi} E_{X 3}} \exp \left[-\rho^{2} \frac{x_{3}^{2}}{E_{X 3}^{2}}\right] \\
& \phi\left(\mathrm{z}_{3}\right)=\frac{\rho}{\sqrt{\pi} E_{\mathrm{z} 3}} \exp \left[-\rho^{2} \frac{\mathrm{z}_{3}^{2}}{E_{\mathrm{z3}}^{2}}\right]
\end{aligned}
$$

After numerical integration, the probability of one hit out of 60 shots in 30 pairs with the TOT method is 0.7175 .
The results of two shooting methods are compared, revealing that TOT brings a much higher probability of one hit out of 60 shots than the traditional method of "solving meeting". From the perspective of error analysis, the TOT method can therefore improve the firing efficacy against the moving targets on the sea.

## 5. TOT SHOOTING METHOD BASED ON AZIMUTH CASCADE FIRING

Cascade firing is a very common shooting method for guns to improve their coverage of target area. In terms of application scenario, cascade firing is classified into distance cascade firing and azimuth cascade firing, which can enhance the respective coverage of shelling in depth and direction in the same way. In this paper, azimuth cascade firing is taken as an example for analysis.

Azimuth effective cascade firing means the effective firing with one scale distance and multiple scale directions. The azimuth cascade firing can greatly damage highly maneuverable targets. In the azimuth cascade firing, the range of scale distance is mainly subject to the observation sensor of the ship, the performance of fire control system, and the maneuver amplitude of the target. When the target moves steadily, the location distribution error of the target at the predicted point depends mainly on the error of the sensor in identifying the target, and the error of the fire control system in determining the elements of the target's motion. For realizable command and correction, three direction scales are normally selected, and the middle scale is corresponding with the target dispersion center. If the TOT method is combined with the azimuth effective cascade firing, the number of falling projectiles is doubled to improve the damage probability to some extent. Through simulation, it is analyzed as follows:

Assuming that a small and fast target on the sea is an unmanned surface vehicle (USV) with 7 m length, 3 m width, 1.5 m height, and 8.5 t displacement. It is 10 km away from the firing ship. It sails at the speed of 30 knots, and in the parallel with the firing ship. The naval gun fire control system calculates the angle probability error $E_{q m}$ and velocity probability error $E_{v m}$ of the target's motion, which are $2.2^{\circ}$ and $1 \mathrm{~m} / \mathrm{s}$, respectively. Based on the relative motion relations, the target's dispersion in the direction is greater than that in the distance. Hence, azimuth cascade firing can achieve better effect.
Subsequently, the model built for large and medium caliber naval guns with the TOT firing method is used to improve the azimuth cascade firing. The improved method is then compared with the traditional azimuth cascade firing in terms of effect.

After consulting with the naval gun operation manual, and performing the simulation analysis with projectile dispersion model, we obtain the comprehensive dispersion error at different distances and the projectile landing point density of both methods as shown in Figures 4 and 5.


Figure 4. Projectile landing point density of the azimuth cascade firing.


Figure 5. Projectile landing point density of the azimuth cascade and TOT firing.
Based on the simulation results, the azimuth cascade firing with TOT can effectively increase the projectile landing point density of large and medium naval guns in maritime activities compared with the traditional azimuth cascade firing. Moreover, its projectile landing point density is approximate to uniform distribution.

As revealed in the above analysis of error and firing efficacy in the TOT shooting method, large and medium naval guns can employ the TOT shooting method based on azimuth cascade firing to improve their cumulative damage probability and achieve higher projectile landing point density within the vicinal area of the target. In other words, the TOT shooting method based on azimuth cascade firing can better strike small and fast targets on the sea regardless of their good maneuverability.

## 5. CONCLUSION

This paper presents a model of large and medium naval guns with the "time-on-target" (TOT) shooting method. Meanwhile, TOT is combined with the practical tactics of azimuth cascade firing for naval guns to verify its feasibility and effectiveness.

This paper provides a new approach for large and medium naval guns to strike small and fast targets on the sea, which help improve the capability of ships against such targets ass unmanned vehicles and armed speedboats.

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