

ADVANCES IN
Sampling Theory
and Techniques

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Preface

Signal sampling is the major method for converting analog signals into sets of numbers that form digital models of the signals. The key issues in the sampling theory and practice are

- What is the minimal amount of numbers, or what is the minimal sampling rate, sufficient to represent analog signals with a given accuracy?
- What kinds of signal distortions are caused by their sampling?
- What signal attributes determine the minimal sampling rate?
- How can one sample signals with sampling rates close to the theoretical minimum?
- Is it possible to resample sampled signals without introducing additional distortions due to the resampling?
- What are adequate discrete representations of signal transforms, such as convolution and Fourier transforms?

All of these issues are addressed in this book, supplemented by MATLAB[®] exercises, which you can download via the following link: http://spie.org/Samples/Pressbook_Supplemental/PM315_sup.zip

Researchers, engineers, and students will benefit from the most updated formulations of the sampling theory, as well as practical algorithms of signal and image sampling with sampling rates close to the theoretical minimum and interpolation-error-free methods of signal/image resampling, geometrical transformations, differentiation, and integration.

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