Teaching Quantum Concepts with Classical Optics

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Abstract: We propose to educate quantum concepts through the analogous classical light context. An optics platform has the advantage of being stable, controllable, visualizable, as well as accessible to pre-college students.

Surprising recent studies have demonstrated that entanglement, a non-separable feature that was conventionally considered to be uniquely quantum, can also exist in classical light fields [1–10]. Growing attentions have been devoted to the field of *classical light entanglement* with more and more analog analyses between quantum concepts and classical optics. Quantum-analog concepts in classical light have been demonstrated to be capable to violate the Bell inequality [2], improve measurement and quantification of polarization [3, 4], control fundamental wave-particle duality [5, 6], and simulate simple quantum tasks [7–9], etc. These encouraging results suggest that classically entangled light can be potentially served as a physical platform to educate fundamental quantum concepts and even to demonstrate the implementation of simple quantum information and computation tasks.

Here we propose to employ the tensor structure of a classical light beam to introduce the quantum concepts of vector spaces, superposition, angular momentum, coherence, entanglement, interference, qubit analogs, quantum information, etc. An optical beam contains three principal degrees of freedom (DOFs), i.e., polarization, spatial property, and temporal property, and can be in general described as

$$\vec{E}(r,t) = \hat{x}G_x(r)F_x(t) + \hat{y}G_y(r)F_y(t), \tag{1}$$

where $\hat{x}, \hat{y}$ characterize the two orthogonal unit polarization directions, $G_x(r), G_y(r)$ represents spatial domain vectors that are spanned by infinite number of basis spatial modes (e.g., the Hermite Gaussian modes as shown in Fig.1), and $F_x(t), F_y(t)$ are temporal domain vectors that can be described by superpositions of basis temporal modes (see illustration in Fig.1).

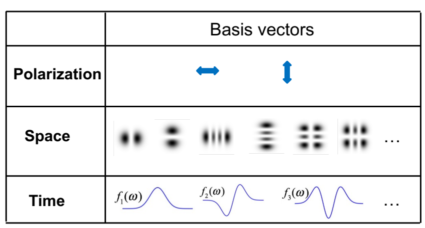

Fig. 1. Tensor structure and basis states of an optical beam with three principal DOFs: polarization, spatial domain, and temporal domain.

To emphasize the vector nature of different DOFs, we adopt the Dirac notation to describe the optical field [3], i.e., $\vec{E}(r,t) \to |E\rangle$, $\hat{x} \to |x\rangle$, $G_x(r) \to |G_x\rangle$, $F_x(t) \to |F_x\rangle$, etc. Then the light beam can be rewritten as

$$|E\rangle = |x\rangle \otimes |G_x\rangle \otimes |F_x\rangle + |y\rangle \otimes |G_y\rangle \otimes |F_y\rangle, \tag{2}$$

This form shows directly the analogy of the mathematical tensor structure of a three-party quantum state.

From the light field (2), one can teach the quantum concept of vector space with either one of the three optical DOFs. Basic linear algebra concepts such as basis states, vector rotation, vector superposition can be easily explained in the optical context, for example with the linear, circular, and elliptical polarization states. Then the combination of two polarized beams, can directly illustrate the idea of coherence and interference which is mathematically exactly the same as the quantum wave functions, where the concept of quantum probability will be replaced with the measurement of the intensity of light.

Sixteenth Conference on Education and Training in Optics and Photonics: ETOP 2021,
edited by A. Danner, A. Poulin-Girard, N. Wong, Proc. of SPIE Vol. 12297, 1229727
© 2022 SPIE · 0277-786X · doi: 10.1117/12.2635581

When two or more DOFs are involved, it will be able to represent the concept of tensor products of vector spaces, as described in (2). The tensor structure of vector spaces resembles that of a multiparty quantum state. When more than one term of tensors are present, it allows the existence of entanglement [10]. Fig. 2 shows an optical setup to create an entangled state $|E\rangle = |x\rangle \otimes |\mathrm{HG}_{10}\rangle + |y\rangle \otimes |\mathrm{HG}_{01}\rangle$, where $\mathrm{HG}_{10}, \mathrm{HG}_{01}$ are first order Hermite Gaussian modes. This provides a directly visualizable picture of how entangled state can be generated and how the two DOFs are strongly correlated.

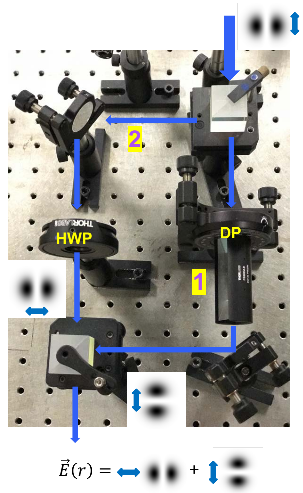

Fig. 2. Setup to generate classical light entanglement between the polarization and spatial DOFs.

With the creation of entanglement with two qubit-like structures, one can then use these two-state vector spaces to represent a quantum bit and realize simulation of quantum tasks. The classical optical beam can then be employed to demonstrate the implementation of simple quantum tasks such as quantum teleportation [7, 8], qubit channel [9], etc.

In summary, we proposed to use the tensor structure of classical optical beams to introduce quantum concepts of vector spaces, superposition, angular momentum, coherence, entanglement, interference, qubit analogs, quantum information, etc., in a visualizable way. This will greatly help the education of quantum science to high school, pre-college, and college students. We acknowledge the support from Stevens Institute of Technology.

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