Chromatic aberrations in lens design

Christiaan H.F. Velzel

Philips CFT, building SAQ
PO Box 218, NL 5600 MD Eindhoven, the Netherlands

Jacob L.F. de Meijere

ASM Lithography
De Run 1110, NL 5503 LA Veldhoven, the Netherlands

ABSTRACT

In texts on geometrical optics and lens design usually two types of chromatic aberrations are discussed: longitudinal and transverse. From basic considerations on first order geometrical optics follows that, for an axially symmetric system there are three paraxial constants. Therefore three, instead of two types of chromatic aberrations can be discerned.

The third, new, chromatic aberration can be called chromatic pupil aberration. We will describe the consequences of this aberration for the colour correction of optical systems, and show that stable chromatic correction requires the elimination of all three chromatic errors. We will give expressions that can be used in the lay-out of optical systems.

In teaching geometrical optics it is necessary to determine the generic aberrations of a system of given symmetry from first principles; our treatment of chromatic aberrations is an example of this necessity.

Keywords: geometrical optics, lens design, colour correction

1. INTRODUCTION

Geometrical optics is a neglected subject in introductory courses in physics nowadays. The consequence is that usually students are kept unaware of some very basic properties of optical systems. It is necessary to find a way of presenting geometrical optics in such a way that it finds a place among other subjects of the curriculum such as mechanics and electricity, and that its beauty and elegance is seen by the students.

Usually geometrical optics is derived from Snell's laws of reflection and refraction. A more fundamental approach would be to begin with Huygens' and Fermat's principle. From these principles, that can be derived from the wave properties of light, it can be shown that there is a characteristic function (eikonal) that describes the properties of optical systems. The eikonal is the optical path along a ray through the system between two carefully defined points.[1]

One of the advantages of using eikonals for the description of optical systems is that the theory of aberrations is put on a solid foundation. In many cases textbooks on general optics, and even some specialised works on geometrical optics, do not consider the correct number of aberration coefficients. This begins already with the well-known Seidel aberrations. In most textbooks one will find five aberrations mentioned, whereas there are six eikonal coefficients of the fourth order (when the system is symmetric). The sixth coefficient can be shown to represent spherical aberration of the pupil. When the position of the object is shifted, this aberration is mixed with the other aberrations. Therefore it is useful to know also the sixth coefficient.

When a symmetric system is perturbed in such a way that its symmetry is broken, in general it will have eikonal terms of uneven as well as even order; also the number of aberration coefficients will be greater than with a symmetric system. Already in the second order there are ten coefficients, leading to astigmatism, anamorphosis, image rotation and shear. The simplest aberrations of symmetric systems are the chromatic aberrations of the first order, arising from the second order eikonal terms. Because there are three second order eikonal coefficients we expect three first order chromatic aberrations. In the textbooks one usually finds two aberrations mentioned: longitudinal and transverse chromatic aberration. We discuss the function of the third coefficient in the following.

2. CHROMATIC FIRST ORDER ABERRATIONS OF THIN LENS SYSTEMS

Thin lens approximations are usually applied in the first stages of the design of optical systems. This is an important phase of the design process, because the thin lens layout decides to a great deal the properties of the final design.

In deriving the chromatic aberrations of thin lens systems we make use of a result of perturbation theory. This says that the change of the eikonal due to a small perturbation of an optical system can be described to first order in the perturbation parameters as the change in the optical path along the unperturbed rays.

Because we consider only first order aberrations we use only the meridional part of the eikonal. For a thin lens in air, with the reference planes in the lens plane, the second order meridional eikonal is given by

$$E = \frac{1}{2\varphi}(L - L')^2 \tag{1}$$

where φ is the power of the lens, and L, L' are direction cosines of a ray before and after refraction. Because φ is proportional to n - 1 (n is the refractive index) the change of the eikonal due to a change of refractive index δn is

$$\delta E = \frac{\delta n}{n-1}\frac{1}{2\varphi}(L - L')^2 \ . \tag{2}$$

With $L - L' = h\varphi$, where h is the height of the ray in the lens plane, we can write

$$\delta E = \frac{\delta n}{n-1}\frac{1}{2}\varphi h^2 \ . \tag{3}$$

For a system of thin lenses we obtain, with obvious notation,

$$\delta E = \sum_i \frac{\delta n_i}{n_i - 1}\frac{1}{2}\varphi_i h_i^2 \ . \tag{4}$$

When we consider two wavelengths this becomes

$$\delta E = \sum_i \frac{1}{2}\frac{\varphi_i}{\nu_i} h_i^2 \tag{5}$$

where ν_i is a partial dispersion. To find the chromatic aberrations we must express h_i in pupil and field co-ordinates. To first order we have

$$h_i = c_i u + d_i w \tag{6}$$

with (angular) field co-ordinates w and (angular) pupil co-ordinates u, see Fig. 1. The constants c_i , d_i can be obtained by tracing two paraxial rays through the system (a marginal ray with $w = 0$, and a chief ray with $u = 0$). Inserting Eq. (6) in (5) gives

$$\delta E = \frac{1}{2}C_1 u^2 + C_2 uw + \frac{1}{2}C_3 w^2 \tag{7}$$

with

$$C_1 = \sum_i c_i^2 \frac{\varphi_i}{\nu_i} \ , \ C_2 = \sum_i c_i d_i \frac{\varphi_i}{\nu_i} \ , \ C_3 = \sum_i d_i^2 \frac{\varphi_i}{\nu_i} \ . \tag{8}$$

Differentiation of δE with respect to u gives the transverse aberration

$$\delta x = C_1 u + C_2 w \qquad (9)$$

consisting of a defocus C_1 and a change of magnification C_2 / t, where t is the distance from the object to the entrance pupil (u and w are object side co-ordinates). In Eq. (9) we cannot see what the function of the third coefficient C_3 is; usually in the textbooks only C_1 and C_2 are discussed. But we see, by interchanging object and pupil, that C_3 causes pupil defocus. The position of the entrance pupil determines the spot geometry in the image plane by selection of rays. A shift of the pupil to a new position t', with new pupil and field co-ordinates u', w' given by (see Fig. 1)

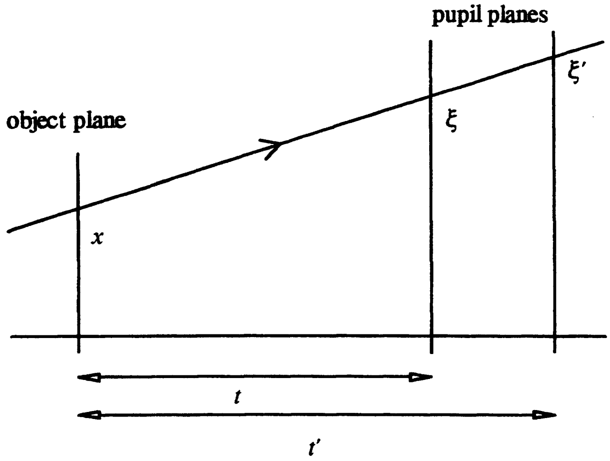

Fig. 1 We define angular pupil and object co-ordinates by $u = \xi / t$, $w = x / t$, and in case of a shifted pupil plane by $u' = \xi' / t'$, $w' = x' / t'$. With these definitions equation (10) of the text follows.

$$u' = u + \frac{t' - t}{t'} w \ , \ w' = \frac{t}{t'} w \qquad (10)$$

leads to a change in eikonal

$$\delta E' = \frac{1}{2} C_1' u'^2 + C_2 u' w' + \frac{1}{2} C_3 w'^2 \qquad (11)$$

with

$$C_1' = C_1 \ , \ C_2' = \frac{t'}{t} C_2 + \frac{t' - t}{t} C_1 \ , C_3' = \left(\frac{t'}{t}\right)^2 C_3 + 2 \frac{t'}{t} \frac{t' - t}{t} C_2 + \left(\frac{t' - t}{t}\right)^2 C_1 \ . \qquad (12)$$

When C_1 and C_2 are made zero, both chromatic errors are corrected for all stop positions. With C_1 and C_2 both non-zero, one can choose the stop position so that lateral chromatic error is corrected. With a shift Δz of the object position, resulting in new pupil and field co-ordinates u'', w'', given by (see Fig. 2)

$$u'' = \frac{t}{t'} u \ , \ w'' = w + \frac{t - t'}{t'} u \qquad (13)$$

where $t' = t - \Delta z$, we have a change in eikonal

$$\delta E'' = \frac{1}{2} C_1'' u''^2 + C_2 u'' w'' + \frac{1}{2} C_3'' w''^2 \tag{14}$$

with

$$C_1'' = \left(\frac{t'}{t}\right)^2 C_1 + 2\frac{t'}{t}\frac{t'-t}{t} C_2 + \left(\frac{t'-t}{t}\right)^2 C_3 \ , \ C_2'' = \frac{t'}{t} C_2 + \frac{t'-t}{t} C_3 \ , \ C_3'' = C_3 \tag{15}$$

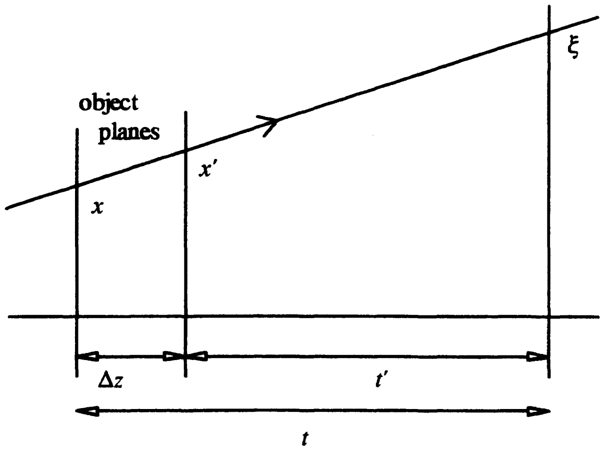

Fig. 2 With the same notation as in Fig. 1, equation (13) of the text follows.

There exists an object position for which the lateral chromatic error is zero, when both C_2 and C_3 and are non-zero. There are in general two object positions for which the longitudinal chromatic error is zero, also when two of the three coefficients C_1 , C_2 and C_3 are non-zero. Stable correction of both chromatic errors for all object positions can be obtained only by making all three coefficients C_1 , C_2 and C_3 equal to zero.

3. DISCUSSION

We showed above that the third aberration coefficient C_3 governs pupil defocus, and must be zero to ensure stable correction of longitudinal and lateral chromatic errors. From the definition in Eq. (7) of three chromatic aberration coefficients it can be seen how to obtain stable correction .

For a lens near the pupil plane d_i is small; for a lens near the object plane c_i is small. The same is true for the optical conjugates of object and pupil planes. Therefore we need lenses at both positions for the correction of chromatic errors. When we divide the lenses in two groups, one group at the pupil for which the c_i are about equal and the d_i are small, and another group near the object (or image) plane for which the d_i are about equal and the c_i are small, we can make a good beginning at chromatic correction by making the sums of φ_i / ν_i equal to zero for both groups. This is the reason that the two groups of a Petzval lens must be corrected independently for colour.[2] With a Cooke triplet with the pupil at the second lens we have $d_2 = 0$ and $d_1 \approx - d_3$. Usually all c_i are positive. Because φ_1 and $\varphi_3 > 0$, lateral colour (C_2) can be corrected easily, but pupil defocus (C_3) cannot.

4. REFERENCES

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