

# Investigating the elasticity of urban rail transit network after disturbance

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## ABSTRACT

In order to improve the elasticity of urban rail transit (URT) network, considering action and recovery process of disruption events, a risk assessment method for URT network based on robustness, vulnerability and elasticity is proposed. By quantifying the overall impact of disturbance on the performance of the URT network, a recovery model is established to maximize the elasticity index of the URT network and an adaptive genetic algorithm is used to solve it. Taking Xi'an URT network as an example, two disturbance scenarios are examined: a random attack and a deliberate attack. The recovery effect of the target recovery strategy, the random recovery strategy and the preferred recovery strategy on the elasticity of the URT network after the disturbance occurs is comparatively analyzed. The results show that the recovery effect of the targeted recovery strategy on the elasticity of the URT network is the best, followed by the preferred recovery strategy. The conclusions of this paper can provide strategic support for the recovery of the URT network after disturbance.

**Keywords:** traffic engineering, urban rail transit, elasticity, network topology, recovery strategies

## 1. INTRODUCTION

Urban rail transit (URT) is the current infrastructure construction that can meet several social development goals. It centralizes scattered travel modes, can increase urban transportation volume and speed and effectively reduce the frequency of car use. Good rail transit network can also reduce multiple pressures in urban centers<sup>[1]</sup>. The urban rail transit system includes subways, light rails, monorails, trams, maglev trains and other modes. The rail transit system is one of the most important functional modules of the city and its network elasticity recovery after disruption is an important guarantee for promoting the safe development of the city<sup>[2]</sup>. Therefore, improving the network elasticity of URT has become an urgent problem to be solved. The data shows that the operating mileage of URT in China will reach 8708 kilometers by the end of 2021<sup>[3]</sup>. The interconnected operation of URT in China not only facilitates the travel of passengers, but also increases the difficulty of operation and various operational disturbance occur from time to time.

Disruptions are natural or man-made disasters that affect the network and eventually lead to station/line function loss, network congestion and other problems. Disruptions in the URT network not only place a huge burden on the urban transportation system, but also have a serious impact on people's lives. On July 20, 2021, a continuous torrential rain in extremely heavy rainstorm caused serious ponding in the Wulongkou parking lot and surrounding areas of Metro Line 5 in Zhengzhou. A train on Line 1 was inundated by floods, resulting in the tragic death of 12 passengers. In March 2021, a sudden equipment failure at Maigaoqiao Station, the first station of Nanjing Metro Line 1, resulted in a 26-hour follow-up trip from Maigaoqiao Station to Line 1. The upward and downward operation of several stations were interrupted, the speed of trains on the entire line was restricted and the interval between departures was extended; Since 2019, the subway network of many cities have been suspended differently in time and space due to the impact of the new crown epidemic on the normal operation of URT<sup>[4-7]</sup>. So It is urgent to carry out research on the elasticity of URT network after disruptions, which is of great significance to improve the rapid recovery of URT network after disturbance.

Currently, research on the elasticity of URT network after disturbance is mainly reflected in the evaluation of network performance after disturbance. Existing studies have mostly used indicators such as robustness<sup>[8]</sup> and vulnerability<sup>[9]</sup> to measure the emergency response capability of URT network. For example, complex network theory is used to analyze

the change of the robustness of the metro network over time in the presence of random disturbance and targeted attacks through simulation<sup>[10]</sup>. Analyze the network robustness of the metro network under different strategies such as node attack, edge attack and overload attack through network utilization<sup>[11]</sup>. Estimate the operational interruption time of the station and characterize the vulnerability of the metro network under disruption events to provide decision support for failure risk management<sup>[12]</sup>. Propose an urban rail network resiliency assessment method based on the elastic triangle model to quantify the performance loss of metro network during disturbance recovery<sup>[13]</sup>. Based on the dynamic risk modeling method of complex network, conduct dynamic risk analysis for urban rail transit system<sup>[14]</sup>.

To compensate for the shortcomings of previous research, this paper proposes to comprehensively consider the network topology, evaluate the elasticity of the URT network by using the recovery rate of the network performance after the disturbance as an index and consider the entire process of the URT network from failure to recovery and finally develop a recovery strategy for the URT network after the disturbance occurs.

## 2. DISTURBANCE

Disturbance include natural disasters such as earthquakes, rainstorms, mudslides, floods and strong winds, as well as man-made disasters such as large-scale events, severe weather, terrorist attacks and traffic congestion.

To comprehensively evaluate the impact of disturbance on the URT network, i.e., the whole process from the occurrence of disturbance to the elimination of their effects, including the action process and the recovery process, the changes in network performance are shown in Figure 1.

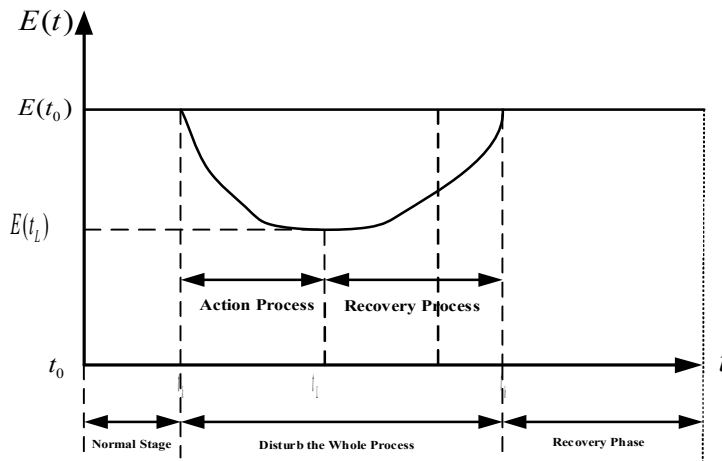


Figure 1. The whole process of URT network after disturbance

As shown in Figure 1,  $t_0$  indicates a certain moment when the URT network is in normal operation,  $t_k$  indicates the moment when the disturbance begins to act,  $t_L$  indicates the moment when the network begins to recover and  $t_h$  indicates the moment when the influence of the disturbance is eliminated.  $E(t_0)$ ,  $E(t_L)$ ,  $E(t_h)$  represent the network efficiency of URT at time  $t_0$ ,  $t_L$  and  $t_h$  respectively.

## 3. EVALUATION METHOD OF URT NETWORK AFTER DISTURBANCE

### 3.1 Robustness and vulnerability evaluation

Robustness is a measure of the ability of a local URT network to withstand shocks and vulnerability refers to the degree to which the system deviates from normal performance under disturbance conditions. Complex network usually use the average shortest path length to measure network accessibility and the calculation formula is as follow:

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij} \quad (1)$$

where,  $N$  represents the number of nodes in the network;  $d_{ij}$  represents the shortest path length between the two nodes.

In order to quantify the difference in the impact of disturbance failures at different stations of the overall function of URT network, the network efficiency index is introduced, denoted as  $E$  and the calculation formula is as follow:

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}} \quad (2)$$

where, the reciprocal of  $d_{ij}$  represents the connection efficiency between the two nodes;  $E$  represents network efficiency.

The calculation formula of robustness is as follow:

$$R_0 = E(t_L) = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d'_{ij}} \quad (3)$$

where,  $d'_{ij}$  represents the shortest path recalculated after deleting the invalid site.

Correspondingly, the vulnerability of URT network can be quantified by the decrease in network efficiency caused by station failure and the calculation formula is expressed as:

$$V = E(t_0) - E(t_L) \quad (4)$$

According to the formula (3-4), the network efficiency and its decline degree of the urban rail network after the disturbance event can be calculated as a measure of robustness and vulnerability. The larger the value of  $R_0$ , the greater the ability of the urban rail network to resist failure and maintain connection efficiency, that is, the better the robustness; the larger the value of  $V$ , the greater the degree of decline in the connection efficiency of the urban rail network due to disturbance failure, that is, the greater the vulnerability.

### 3.2 Elasticity evaluation

Elasticity refers to the overall performance of the system from damage to return to normal within a given period of time. Therefore, the elasticity of the system is measured by the degree of cumulative loss of system performance after a disturbance.

The calculation formula of elasticity is expressed as:

$$T_E = \frac{\int_{t_1}^{t_2} [E(t)] dt}{(t_h - t_k)E(t_0)} \quad (5)$$

where,  $T_E$  represents the elasticity index,  $E(t)$  is the network performance curve, which represents the function of network performance over time,  $t_k$  is the starting moment of disturbance failure,  $t_h$  is the moment when the network performance returns to the initial state.

## 4. RECOVERY STRATEGIES FOR RAIL TRANSIT NETWORK DISTURBANCE

### 4.1 Recovery model after disturbance

This paper proposes a target recovery strategy based on maximizing the elasticity index. The objective function is as follow:

$$\begin{aligned}
\max(T) &= \max[T(X | t)] \\
&= \max \left\{ \frac{\int_{t_k}^{t_h(x)} E(x, t) dt}{[t_h(x) - t_k] E(t_0)} \right\} \\
s.t. \quad &t_h < t_i \leq t_k, i = 1, 2, \dots, s \\
&t_h - t_k = s \times S \\
E(t) &= E(t_i), t_i \leq t < t_i + S
\end{aligned} \tag{6}$$

where:  $[T(X | t)]$  is the elasticity of the URT network after the disturbance,  $X$  is the set of recovery schemes,  $t_2(x)$  is the moment when the network is completely restored when the recovery scheme  $X$  is adopted,  $x \in X$ ,  $E(x, t)$  is the elastic performance of the urban rail transit network at  $t$  using scheme  $x$  after the disturbance occurs,  $t_i$  is the repair completion time of the  $i$  station,  $s$  is the number of failed stations,  $S$  is the time required for each station repair.

## 4.2 Model solving

In this paper, the adaptive genetic algorithm is used to solve the URT network recovery model after the disturbance with the aim of maximizing the elasticity index. The steps are as follows:

**Step 1 Coding:**For the problem of recovering a failed station, the chromosome is divided into segments using the integer coding method, where each segment is the number corresponding to the failed station.

**Step 2 Fitness function:** The individual is the recovery plan of the failed station and the group is the set of recovery plans for the failed station. In this paper, the objective function of maximizing the elasticity index is chosen as the fitness function and the fitness value is the elasticity index.

**Step 3 Selection:** In this paper, the method of combining the optimal conservation strategy and the roulette wheel selection method is used for selection.

**Step 4 Crossover:**This paper uses an adaptive crossover probability function to calculate the crossover probability, that is:

$$p_c = \begin{cases} a_1 + \frac{u_1(T_{\max} - T_1)}{T_{\max} - T_{avg}}, T_1 \geq T_{avg} \\ \frac{u_2(T_{\min} + T_{avg})}{T_1 + T_{avg}}, T_1 < T_{avg} \end{cases} \tag{7}$$

where:  $p_c$  is the self-adaptive crossover probability,  $T_{\max}$  is the maximum value of the elasticity index obtained in the collection of recovery schemes for each generation of failed stations,  $T_{\min}$  is the minimum value of the elasticity index obtained in the collection of recovery schemes for each generation of failure stations;  $T_{avg}$  is the the average value of the elasticity index is obtained from the set of recovery schemes for the failed stations,  $T_1$  is the largest elasticity index in the pair of recovery schemes to be crossed,  $a_1$ ,  $u_1$  and  $u_2$  are adaptive control parameters, which take values at (0,1).

**Step 5 Mutation:**Mutation probability is also one of the key factors affecting the performance of the genetic algorithm. The mutation probability is calculated using the adaptive mutation probability function, ie:

$$p_m = \begin{cases} a_2 + \frac{u_3(T_{\max} - T_2)}{T_{\max} - T_{avg}}, T_2 \geq T_{avg} \\ \frac{u_4(T_{\min} + T_{avg})}{T_2 + T_{avg}}, T_2 < T_{avg} \end{cases} \tag{8}$$

where:  $p_m$  is the adaptive mutation probability,  $T_2$  is the elasticity index of the current timing scheme to be mutated and restored,  $a_2$ ,  $u_3$ ,  $u_4$  are the adaptive control parameters, which take values at (0,1).

## 5. CASE STUDY

### 5.1 URT network topology construction

By the end of March 2023, a total of 8 subway lines have been opened in Xi'an, with a total of 174 stations and an operating mileage of 279 km. The Space-L method is used to construct the distance-weighted topological network model of Xi'an subway, as shown in Figure 2.

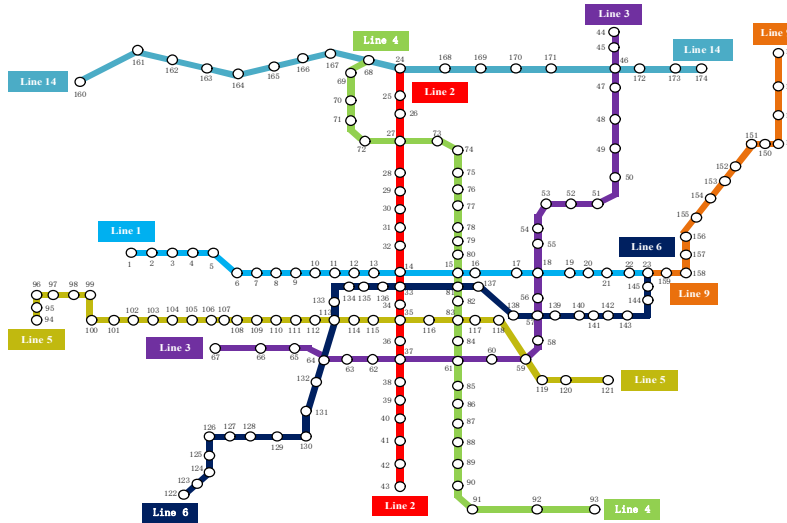


Figure 2. Topological network model of metro rail transit in Xi'an

The operating hours of rail transit in Xi'an are from 6:00 to 24:00. In this paper, the morning rush hour (7:30~8:30) of Xi'an rail transit stations from February 20 to 26, 2023 is used to identify 174 railroad stations. The inbound and outbound passenger flow in the morning peak hour of the station for 7 consecutive days to obtain the OD passenger flow in the morning peak hour between 174 stations.

In the absence of data on the actual occurrence of disturbance, this paper uses computer simulation to determine the impact of the occurrence of disturbance. Assuming that all disturbance occur at 8:00am, random and intentional attacks are used to simulate network failure due to equipment failure and terrorist attacks. When simulating an equipment failure, the number of the failure site is randomly determined; when simulating a deliberate attack, the number of the failure site is determined according to the passenger volume at the morning peak. The failure scenarios simulated in this way are shown in Table 1.

Table 1. Simulated failure scenarios

Random attack failed site	Deliberate attack failed site
2,6,27,39,52,72,85,113,119,157	14,15,18,24,27,33,35,59,64,83

### 5.2 Disturbance risk assessment

Looking at the failure of stations in the Xi'an metro network, the risk assessment is performed using the two dimensions of robustness and vulnerability. Calculate the robustness, vulnerability and elasticity of the Xi'an subway network when a station fails according to the formulas (3 and 4), rank the robustness from small to large, select the top 5 stations and analyze the results as shown in Table 2.

Table 2. Robustness, vulnerability and elasticity analysis of disturbed URT network

Station	Node degree	$R_0$	$V/E$ %	$V$	$V/E$ %	$T_E$
27	4	0.0912	88.54%	0.0080	7.77%	0.9168
18	4	0.0917	89.03%	0.0074	7.18%	0.9117
24	3	0.0918	89.13%	0.0074	7.18%	0.9186
35	4	0.092	89.32%	0.0073	7.09%	0.9207
14	4	0.0922	89.51%	0.0070	6.80%	0.9210

As shown in Table 2, the robustness of the metro network is worst after the failure of Station 27, i.e., the main administrative station, with a robustness index of 0.0912 and a vulnerability index of 0.0080. At this time, the efficiency of the network is only 91.68% of the normal operating condition, which is a decrease of 8.32%. In addition, most of the best rated stations are interchange stations with a node degree of 4. The failure of these stations has great impact on the network efficiency and the ability to withstand failures and maintain the connection efficiency of the remaining normal stations is lower, so special attention is required.

### 5.3 Recovery strategies after disturbance occurs

For the convenience of solving, this paper assumes that the URT network is disturbed at time 0, that is,  $t_k = 0$  and the repair time of each station is 1h. According to formula (6-8), during the evolution process, based on the relationship between the elasticity index of the recovery time series scheme and the average elasticity index of all recovery time series, dynamically adjust the probability of crossover and mutation.

As shown in Table 3, according to formula (1-5), calculate the network elasticity resulting from using random, recovery, preference recovery and target recovery strategies under the two disturbance simulation scenarios and compare and analyze the station repair sequence and network elasticity resulting from different recovery strategies under the two simulation scenarios. In this paper, two types are selected based on the node degree and station ridership.

Table 3. Analysis of URT network elasticity recovery sequence in two scenes

Scene	Recovery strategy		Station recovery order	elasticity
Random attack	Random recovery		52→39→2→113→72→27→6→157→85→119	0.8743
	Preference recovery	Node degree	27→113→6→39→72→85→119→52→157→2	0.8998
		Station traffic	39→85→6→113→27→52→119→72→2→157	0.9002
	Target recovery		27→39→85→113→6→52→119→2→157→72	0.9197
Deliberate attack	Random recovery		18→14→64→35→24→15→27→83→33→59	0.8572
	Preference recovery	Node degree	24→27→14→33→35→18→15→64→59→83	0.8685
		Station traffic	64→35→14→18→59→15→27→33→24→83	0.8707
	Target recovery		64→35→59→18→15→14→27→33→24→83	0.9024

Table 3 shows that for the two disturbance scenarios, the target recovery strategy has the best recovery effect for the rail transit network, followed by the preferred recovery strategy. It can be seen that the target recovery strategy proposed in this paper can minimize the performance loss of the URT network. It can be seen that the repair order of the failed station is the key to the elasticity of the URT network and the elasticity of the URT network achieved by choosing different repair orders is also different.

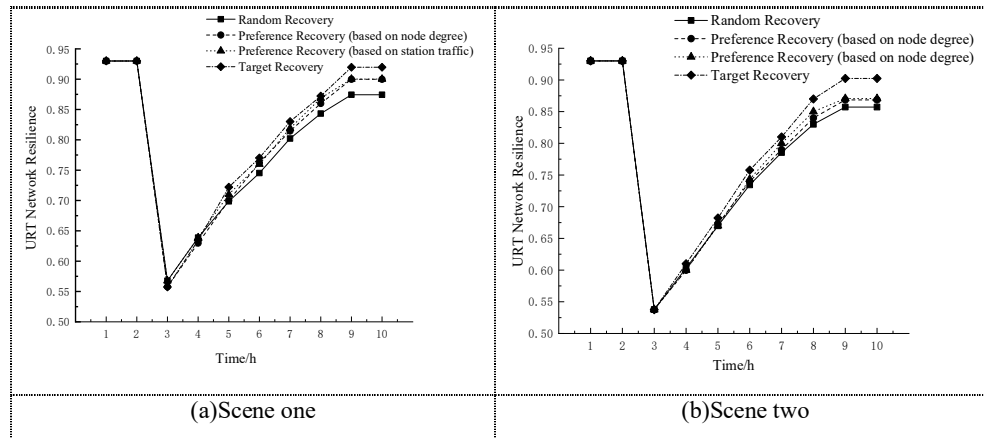


Figure 3. URT network elastic recovery curves under two disturbance scenarios

Figure 3 verifies that the repair effect of the target recovery strategy is always the best with the passage of repair time under two disturbance occurrence scenarios, but there are differences in the repair effect of the two preference recovery strategies under different disturbance scenarios. Therefore, when choosing the order to repair damaged stations, not only should consider the damaged stations, but also the impact of damaged station passenger flow on network performance should be considered.

## 6. CONCLUSION

The main conclusions are as follows:

- (1) This paper defines the disruption and quantitatively analyzes the elasticity of the URT network and proposes a risk assessment method for the disturbance and then proposes a target recovery strategy model for the URT network after the disturbance and gives a solution method and validated with the Xi'an metro network.
- (2) In the two disturbance scenarios with random and intentional attacks, the target recovery strategy has the best recovery effect for the URT network, followed by the preference recovery strategy and the random recovery strategy has the worst recovery effect.
- (3) This paper describes the optimal recovery strategy after the failure of the disrupted URT station. The target recovery strategy can improve the elasticity index of the URT network, reduce the performance loss of the network and provide support for recovery decisions after the disturbance occurs. In the next step of the research, the recovery time and recovery cost can be further considered and the specific details of the recovery time and recovery cost can be further explored based on the determination of the optimal recovery sequence of the URT network.

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