Review of Research on Vehicle Routing Problems

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ABSTRACT

With the rise of the social logistics industry, the vehicle routing problem has always been a hot topic for scholars to study. This paper focuses on the recent research advances in the last 5 years of the vehicle routing problem, which contains the basic vehicle routing problem (VRP), vehicle routing problem with capacity constraints (CVRP), vehicle routing problem with time windows (VRPTW), split delivery vehicle routing problem (SDVRP), dynamic vehicle routing problem (DVRP), other types of VRPs, and so on. By subdividing the above according to VRP characteristics and describing its solution method. Finally, future research trends and development trends of VRP are summarized.

Keywords: VRP, Transport paths, Solution methods, Future vehicle path trends

1. INTRODUCTION

With the development of society and the prosperity of logistics industry, logistics distribution plays an increasingly important role in residents’ lives. Optimization of transportation route is an indispensable part of logistics distribution, which has a great impact on profit, cost, speed, and time of freight transportation. Hence, vehicle routing problem (VRP) should be considered in the transportation route planning. VRP refers to designing reasonable delivery routes for vehicles to transported goods from distribution centers to customers under a set of constraints e.g., vehicle capacity, transportation time, transportation demands so as to obtain one or multiple goals, e.g., minimum costs, minimum time, minimum delay (see Figure 1). Exact algorithms and heuristic algorithms are two common categories of algorithms used to solve VRP (see Figure 2). Since VRP was first proposed in 1959, its variants have attracted increasing attention of researchers. These researches highlight the necessity of an up-to-date review of the literature in VRP.

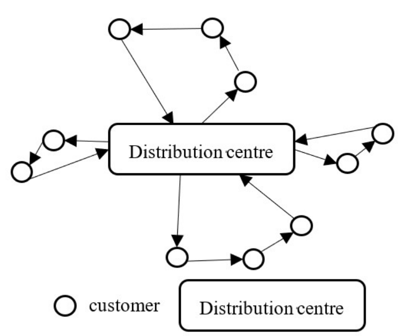

Figure 1. Explanation of VRP

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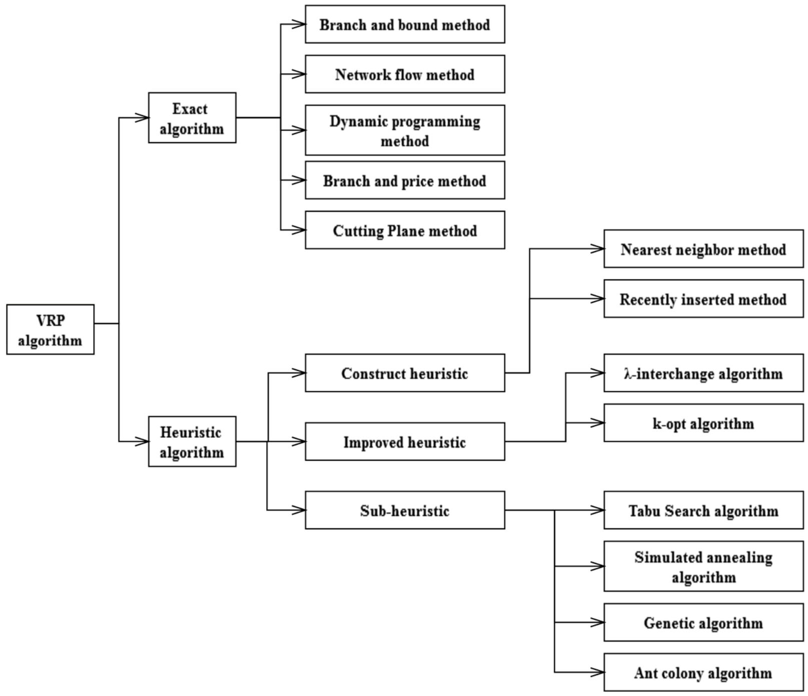

Figure 2. Classification of algorithms to solve VRP

2. CAPACITY CONSTRAINED VEHICLE ROUTING PROBLEM (CVRP)

CVRP is prevalent problem considering vehicle load constraints, which are basic implicit constraints. Generally, in order to obtain the minimum total cost and the shortest distance, the interaction between the number of vehicles, cargo distribution and route planning also needs to be taken into account in CVRP. Currently, heuristic algorithms are mainly used for solving CVRP. Ant colony algorithm (ACA) is a typical heuristic algorithm for solving CVRP, which is an effective method used to solve complex combinatorial optimization problems [1, 2]. The previous studies reveal that although ACA can find a feasible solution within a certain time, it is limited to small scale problems. Hence, a few of researchers have enhanced ACA when solving CVRP. Yan et al [3] propose an algorithm based on maximum and minimum ant system (MMAS), which adds some appropriate constraints to the pheromone of ant systems and strengthen the mining of information of elite ant paths so as to improve the optimization criteria ability of ACA. Gao et al [4] propose a hybrid algorithm, which integrates fireworks algorithm (FWA) and ACA, which increases the diversity of algorithms so as to find more locally optimal vehicle routes. In addition to the above modified heuristic algorithms, some new Intelligent algorithms also have been applied to solve CVRP. Xia et al [5] apply an artificial bee colony algorithm based on adaptive large neighborhood search, which can effectively solve the problems of poor convergence, insufficient exploration ability and long solution time when solving CVRP. Yuan et al [6] put forward an adaptive simulated annealing and artificial fish swarming algorithm (A-SAAFSA) for the optimization of vehicle routing from multiple warehouses to multiple costumers.

3. VEHICLE ROUTING PROBLEM WITH TIME WINDOW (VRPTW)

VRPTW sets the earliest time and latest time for the beginning of delivery service for each customer point (demand point) and finish the delivery within its time window. VRPTW can be classified into three categories, i.e., VRP with hard time window (VRPHTW), VRP with soft time window (VRPSTW), and VRP with combined time window (VRPCTW). In the existing researches, genetic algorithm, tabu algorithm, simulated annealing, improved particle swarm algorithm, and ant colony algorithm are the main methods to solve VRPTW. Shen et al [7] design a hybrid swarm intelligence algorithm that combines ACA and brain storm optimization algorithm (BSOA) and prove the effectiveness of this hybrid algorithm to solve VRPTW. VRPHTW assumes that if a vehicle arrives earlier than the time window, it must wait, and if a vehicle arrives later than the time window, it will be rejected. Fang et al [8] use a choice column generation algorithm to solve large-scale VRPHTW. VRPSTW assumes that it is not necessary to serve customers point within the time window, but if it a vehicle arrives outside the time window, it will receive a penalty. Tang et al [9] consider that mechanical failure, weather conditions and other stochastic factors in the transport process are prone to have impacts on various modes of transportation, and use crow search algorithms to confirm that VRPSTW has more practical significance. Li et al [10] establish a soft time window and hard time window distribution route optimization model considering the difficult and cost in the delivery process, and use genetic algorithms to solve the model with a one-stage solution method, which validate the effectiveness of proposed methods. Tong et al [11] use C-W saving algorithm to satisfy the requirements of soft and hard time windows of customers and provide reference for distribution decisions of logistics enterprises. In addition to the above classic VRPTW, many researches have also investigated the variants of VRPTW including VRP with time window and simultaneous pickup and delivery (VRPSPDTW), VRP with fuzzy demand multiple trips considering time window preferences (MVRPFDTW), VRP with fuzzy time windows (VRPFTW), VRP with stochastic demand and time windows (VRPSDTW), VRP with service selection and time windows (VRPSCTW), and multi-vehicle ranges (multi-distribution centers) VRP with time window (MDVRPTW). VRPSPDTW not only consider time window constraints but also involves with picking up and delivering goods at customer points and return the goods to the distribution center afterwards. In VRPSPDTW, routes are rationally planned to minimize total distribution cost. Currently, there is relatively few studies in VRPSPDTW. Jia et al [12] use genetic algorithm for solving a two-level VRPSPDTW. The heuristic algorithms for solving VRPSDPTW problem are shown in Table 1.

Table 1. Heuristic algorithms for solving the VRPSDPTW problem

Author	Solution method	Problem solving
Zhou [13]	A decomposition-based local search algorithm	Large Scale Many-Objective VRP for Simultaneous Delivery and Pickup and Time Windows
Öztaş [14]	Hybrid algorithms combining iterative local search, variable neighborhood descent and threshold acceptance metaheuristics	VRPs for delivery and pick-ups
Hornstra [15]	Adaptive Large Neighborhood Search (ALNS) Metaheuristics	VRP with simultaneous pickup delivery and handling costs (VRPSPD-H)

3.1 Multi-trip VRP with consideration of fuzzy demands and time window preferences (MVRPFDTW)

MVRPFDTW refers to the determination of multi-trip route planning for vehicles considering customers' time window preferences on the basis of fuzzy demand VRP. Zhang et al [16] study the fuzzy demand problem considering time window preference by breaking the one-way restriction of vehicles and introducing customers' time window preference in order to adapt the fuzzy demand VRP to the reality of multi-trip vehicle routing. Neira et al [17] build two integer planning (IP) models which are used for multi-trip VRPs including time windows, service-related loading times and finite trip durations.

3.2 Fuzzy time window with VRP (VRPFTW)

VRPFTW has uncertain service time to customer points. Qamsari et al [18] propose a novel approach to the inventory routing problem for fuzzy time windows considering customer satisfaction and present a multi-priority structure for vehicles to visit customers. Li [19] considers the characteristics of fresh food delivery to develop a multi-objective vehicle routing optimization model with fuzzy time window for fresh food delivery, and designs an improved ACA based on rolling time-domain delay distribution to solve the problem according to the idea of the single-order distribution to get the optimal combination of the distribution cost and the customer satisfaction. Diao et al [20] propose a hybrid genetic algorithm for multi-depot open VRP with fuzzy time windows (MDOVRPFTW) without maximum time windows for a pharmaceutical logistics company in Beijing.

3.3 VRP with stochastic demands and time windows (VRPSDTW)

The latest relevant researches on VRPSDTW are as follows. Wang et al [21] consider a bi-objective optimization problem with stochastic demands so as to efficiently balance operational cost and customer satisfaction, and propose a new evaluation method to measure customer satisfaction affected by time windows. They combine the NSGA-II algorithm with adaptive large neighborhood search to minimize the expected operational cost of a given route under the vehicle co-operation strategy by a dynamic programming algorithm. De La Veg et al [22] propose an integer L-shaped algorithm, which is also the first exact method tailored for VRPSDTW. Wang et al [23] develop a heuristic algorithm based on improved ant colony optimization (IACO) and simulate annealing (SA) to solve a periodic VRP with time window and service choice (VRPSCTW) problem. Comparison analysis with other algorithms is used to verify the performance of the proposed hybrid algorithm.

3.4 Multi-Depot VRP with Time Windows (MDVRPTW)

Yesodha et al [24] solve MDVRPTW using improved firefly algorithm (IFA) and prove the novelty and feasibility of the algorithm. Bezerra et al [25] propose a smart general variable neighborhood search (SGVNS) algorithm for solving MDVRPTW. Since VRPTW must take into account the service time at the customer point, the total cost of the VRPTW includes transportation cost, time cost, time penalty cost, etc.

4. SPLIT DELIVERY VEHICLE ROUTING PROBLEM (SDVRP)

4.1 Progress in the study of SDVRP

Currently, existing exact and heuristic algorithms make it difficult to perform balanced calculations in terms of time and degree of optimization. When solving large-scale SDVRP with exact algorithms, heuristic strategies are usually employed to help to speed up the solution. When solving large-scale SDVRP with heuristic algorithms, both metaheuristics and hybrid heuristics are commonly used.

In terms of exact algorithms, Gouveia et al [26] develop the first exact algorithm and propose an integer programming formulation using a small number of decision variables and several sets of valid inequalities, which is relatively simple but can effectively compute high-quality initial solutions and is tested in the multi-depot split-delivery VRP (MDSDVRP). Gschwind et al [27] propose a branch-and-price and cut algorithm to solve the commodity-constrained split distribution VRP (C-SDVRP) with lower bounds. As shown in Table 2, heuristic algorithms are summarized to solve the SDVRP problem.

Table 2. Solving SDVRP with heuristic algorithms

Heuristic algorithm	Algorithms
hybrid heuristic algorithm	Local search algorithm for routing and a genetic algorithm and several construction heuristics for packing [28]
	An enhanced neighborhood search algorithm (ENS) incorporated with the maximum-space-utilization-based tabu search packing algorithm (MSUTS) [29]
	Q-learning theory and differential evolutionary algorithms for designing memetic algorithm [30]
Sub-heuristic	Novel genetic quantum algorithm [31]
algorithm	Three ant colony algorithms [32]
	Memetic algorithm [33]
Generic heuristic algorithm	Adaptive large neighborhood search (ALNS) [34]
	Branch and Cut Algorithm [35]

4.2 Progress in research on the type of derivation of SDVRP

According to characteristics and constraints, there are various types of SDVRP. There are many methods for solving SDVRPTW, such as taboo search algorithms, column generation methods, branch and cut algorithms ...... etc.; there are also many methods for solving pickup and delivery VRP (PDVRP), such as two-stage heuristic methods, competitive decision algorithms, neighborhood search heuristic algorithm ...... etc. In practice, in order to avoid increasing vehicle emptying rates and wasting transport resources, transportation companies will triage customer demands when providing services at task points where demand is likely to be high.

5. DYNAMIC VEHICLE ROUTING PROBLEM (DVRP)

Most of the previous literature on VRP assumes that customer demands and vehicle travel time are static factors, which do not change during route planning and implementation. However, there may be some dynamic factors in reality. Changes in vehicle routes caused by customers' changing needs, weather conditions, traffic conditions, personnel, vehicles and other factors may bring a lot of uncertainties, so it is necessary to arrange vehicle routes according to real-time dynamic information. With the rapid development of artificial intelligence and intelligent transportation networks, real-time dynamic demands and real-time traffic information networks are accessed easily. DVRP can receive various random and uncertain dynamic information in real time, which can update vehicle routes accordingly. The most important feature of DVRP is the real-time reception of random and uncertain dynamic information about the demands of new and old customer points, adjustment of service time, and the situation of traffic congestion.

5.1 Classification of DVRP

Elements that affect DVRP include: dynamic demand, dynamic road travel time, and dynamic service. Based on the elements, DVRP can be classified into three types, i.e., dynamic demand based VRP (DDVRP), real-time traffic information based VRP (RTVRP), and dynamic demand and real-time traffic information based VRP (DDRVRP). DDVRP refers to changes in delivery services, delivery locations, and time windows for new and existing customers, resulting in vehicle routes being updated based on dynamic demand information. RTVRP refers to traffic congestion caused by unexpected events such as vehicle breakdowns, traffic accidents, weather changes, etc., which results in the vehicle travelling speed being deeply affected by the real-time traffic conditions, and the vehicle needs to be updated in real-time during the process of updating the traffic information and travelling routes. The combination of the above two problems is DDRVRP, which is the most difficult and complex problem in DVRP.

5.2 Progress in the study of DVRP

Currently, several researchers have proposed new models and theories to address the real-time nature and complexity of DVRP. Abdirad et al [36] propose a hierarchical approach consisting of three stages, i.e., clustering, route construction, and route improvement to solve large-scale DVRP (LSDVRP), and their results prove the applicability of the proposed approach. Pan et al [37] propose a novel deep reinforcement learning framework to solve dynamic and uncertain VRP (DU-VRP), whose objective is to meet the uncertain servicing needs of customers in dynamic situations.

5.3 Research on DDVRP

Researches on DDVRP can be classified into VRP with single dynamic demand element and VRP with multiple dynamic demand elements based on the number of dynamic demand elements. The single dynamic demand element is mainly based on new customer demand and is studied with the optimization objective of minimizing the total composition cost by considering constraints such as time window, capacity, and multiple cycles. VRP with multiple dynamic demand elements mainly considers multiple dynamic demand factors such as new customer demands, customer cancellations, demand changes, time window changes, and changes of customer or vehicle location.

5.4 Research on RTVRP

Existing literature on RTVRP mainly focuses on capacity constraints, RTVRP with time window constraints, and RTVRP with joint constraints on capacity and time window, as shown in the following table.

Table 3. Classification of RTVRP studies

Research direction	Optimization objectives	Methodology
RTVRP with capacity constraint	Addressing VRPs caused by congestion [38]	A multiple colonies artificial bee colony algorithm
	Route electric vehicles to meet all shelter needs within their time window and minimize total vehicle fleet driving costs [39]	A two-stage approach
RTVRP with time window constraint	Optimizing automated replenishment platform, network communication method and coordinated distribution optimization technology [40]	A hybrid algorithm of genetically improved set-based particle swarm optimization (S-GAIPSO)
	Minimizing the overall distance traveled by all the vehicles across all depots given the capacity constraints [41]	Quantum annealing (QA)approach
RTVRP with joint constraints on capacity and time window	Minimum cost [42]	Optimization algorithm
	Minimizing the energy consumption [43]	A new hybrid evolutionary algorithm

5.5 Research on DDRVRP

DDVRP is one of the most complex forms of VRP problems. The research is summarized as shown in the table below.

Table 4. Study of the DDRVRP problem

Research direction	Optimization objectives	Methodology
Customer needs and Real-time traffics	Improving the accuracy of time-cost estimation [44]	Simulated Annealing (SA) algorithm
Real-time demands and emergencies	Reduced transport costs and distances [45]	SFSSA algorithm
dynamic passenger flow	optimizing the tracking interval between adjacent trains [46]	Coyote Optimization Algorithm (COA)
DVRP with capacity and time windows	To effectively address DVRP issues [47]	A new strategy for incorporating the Improved Firefly Algorithm (IFA) into the framework of Spiked Neural P (SNP) systems

5.6 Progress in research on solution methods for DVRP

Route updating strategies and optimization algorithms are usually used to solve VRP. Four types of routes updating strategies, i.e., periodic updating, dynamic event updating, customer point updating, key point updating are mostly employed. Existing optimization algorithms for DVRP mainly are heuristic algorithms. Currently, researches on DVRP are based on the combination of capacity constraints, time window constraints, and dynamic information such as dynamic demand elements and dynamic travelling time, which lays the foundation for DVRP research. As shown in Table 5, a review of heuristic algorithms on the study of DVRP.

Table 5. Overview of heuristic algorithms on the study of DVRP

Types of algorithms	Problems solved	Specific algorithm
heuristic algorithm	DVRPSTW	An improved RTR algorithm and plug-in real-time path planning algorithm [48]
	DDVRP	NSGA-III [49]
	DDRVRP	Simulated Annealing (SA) algorithm [44]
	DVRP	Parallel Sparks Genetic Algorithm [50]
	RTVRP	A two-stage approach [39]

6. CONCLUSION AND OUTLOOK

As a classic combinatorial optimization problem, vehicle routing has always been a hot and difficult problem, whether it is in emergency management or logistics distribution, it is very important to its reasonable planning. This paper summarizes the basic features of VRP and its research progress in recent years, and classifies the variants of basic VRP. Additionally, we also review the research progress of exact algorithm, heuristic algorithm and machine learning algorithm in solving VRP.

Despite the wide range of studies in VRP, the following issues should be further addressed:

(1) Develop more efficient and accurate algorithm. When the heuristic algorithm is used to solve VRP and its variants, it often faces two disadvantages, which are easy to fall into a locally optimal solution and slow convergence speed. The previous work either enlarging the search space or using adaptive factors to control the algorithm will increase the complexity of the algorithm. So how to develop algorithms to avoid locally optimal solution and slow convergence rate is still a problem that needs to be paid attention to.

(2) Consider carbon emission into VRP. With the attention to ecological environment protection and the concern about resource scarcity, green VRP is the direction and focus of future development. At present, some literatures have added carbon emission constraints to VRP research. Electric vehicles with low energy consumption and no pollution to the environment are considered to extend the classical VRP, and the constraints of limited vehicle driving range and vehicle charging location also should be considered.

(3) Focus on joint routing problem of new transportation tools and vehicles. The application of new transportation tools such as unmanned vehicles, drones and delivery robots to logistics distribution has led to new VRP. Due to their strong autonomy, autonomous vehicles and delivery robots need to consider uncontrollable factors such as traffic conditions and loading conditions in VRP, so there is a problem of system coordination, which is also a new research direction.

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