Optical correlator as a tool for physicists and engineers training in signal processing

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ABSTRACT

In many fields of Physics and Engineering the linear systems are studied. The Fourier Transform is a powerful tool for analyzing their behaviour in terms of the frequency contents, both for the input signal and the output signal. The concept of Fourier Transform is generally introduced by mathematical tools. An optical correlator is a set-up that allows to display the decomposition of a signal (1D or 2D) and the processing of this signal. In this communication we use an optical correlator with two arms that gives the display of the Fourier plane and the final plane simultaneously. In the first arm, we can visualize the decomposition of the signal in the Fourier space with the application of a given filter. The effect of the filter on the signal is observed in the second arm. The detection is performed by means of CCD cameras and displayed on the computer monitor. Binary filters help to understand the frequency contents of a signal by substraction of frequencies. Gray level filters and complex valued filters allow the synthesis of any transfer function. In particular we show the application to pattern recognition.

Keywords: Fourier Transform, Image Processing, Optical Correlator, Linear System

1. INTRODUCTION

Optical Signal Processing deals with two dimensional functions: the images. These two dimensions are the spatial coordinates that locate the value of brightness of the image at a particular point. The information content of the image can be visualized in two different ways: the direct space and the frequency space. Mathematically the Fourier Transform connects both points of view[1]. The important point is that optically, by taking profit of the diffraction properties of light it is very simple to visualize the image in any of the two spaces. In each space different properties of the image are emphasized, and in many tasks of optical signal processing the frequency space serves to operate on the image in a simpler and more intuitive way than it would be done in direct space.

Optical Signal Processing provides us with a visual way of learning different concepts of the general theory of Signal Processing, that is both applied for Optical and for Electrical Signal Processing[2]. The latter deals with one dimensional signals (time-dependent signals) while in Optics the functions are two dimensional (spatial-dependent signals). Nevertheless, the mathematical structure is analogous for both cases. One of the advantages of Optical Signal Processing versus Electrical Signal Processing is the inherent Fourier transforming properties of light propagation[3] that helps the student to directly visualize the results of the signal processing just making use of a light source and some lenses. In conclusion, taking into account the educational point of view "an image is worth a thousand words".

Due to the wave nature of light, when light is perturbed by any kind of inhomogeneity in its way of propagation the result is that the wavefront is modified in some way by this perturbation. We say that the wavefront has been diffracted by the inhomogeneity, that may consist in a modulation of the absorption or in a modulation in the phase in the light beam. Normally, Scalar Diffraction Theory is enough to calculate the perturbed wavefront[4]. Making use of the Scalar Diffraction Theory we find that applying the Fraunhofer approximation we obtain the Fourier Transform of the inhomogeneity in a certain plane (in the infinite when it is illuminated by a plane wave). Nonetheless, the use of lenses approximates to a finite distance this Fourier plane.

An optical correlator is a very powerful information processing system. The information is processed in parallel and at the speed of light. In a correlator a series of lenses projects the Fourier Transform of an input image at specific distances. At the same time different processing operations can be applied to this input image. In Section 2 we are going to explain in more detail the way a correlator works. This will help to explain the possibilities that it provides in Signal Processing Education. In Section 3 we describe the optical correlator with two arms that we use in our students laboratory. Different examples of the possibilities of the optical correlator are presented in Section 4. The application to Optical Pattern Recognition is discussed in Section 5.

2. CONVERGENT CORRELATOR

We are specially interested in the Vander Lugt convergent correlator[5], shown in figure 1. The Vander Lugt converging geometry is a variant of the classical 4f correlator. The system is illuminated with a monochromatic point source O. Lens L1 with focal length F_1 produces a converging wave that gives the image O' of the point source. If an optical transparency with transmission $f(x, y)$ is inserted behind the lens, the complex amplitude $A(u, v)$ in the Fourier plane is given by

$$A(u, v) = C \exp\left(i \frac{\pi}{\lambda D_3} (u^2 + v^2) \right) F\left(\frac{u}{\lambda D_3}, \frac{v}{\lambda D_3} \right) \tag{1}$$

where C is a constant, (u, v) represents the spatial coordinates at the Fourier plane, λ is the wavelength of the light, D_3 is the distance between the scene transparency and the Fourier plane, and the function $F(\mu, v)$ is the Fourier transform of the function $f(x, y)$.
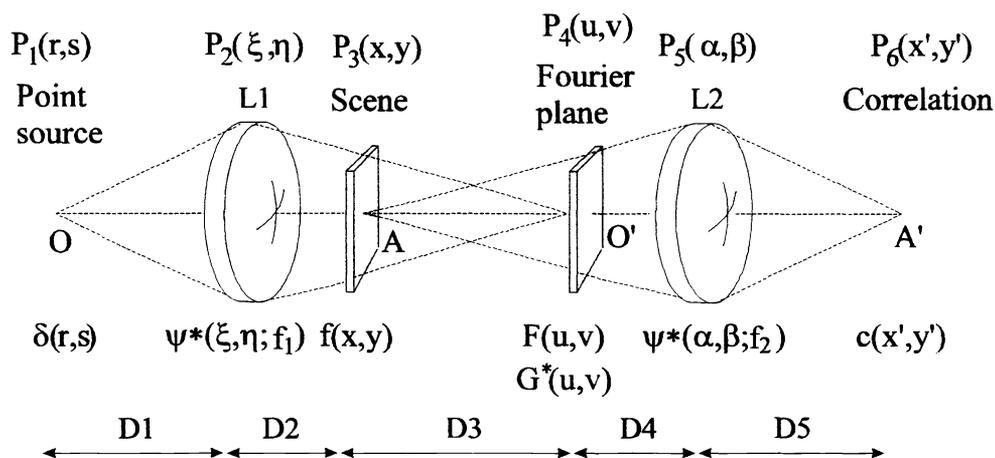

Fig.1. Geometry of the optical convergent correlator. L_1 and L_2 are convergent lenses.
O' is the image of O given by L_1, and A' is the image of A given by L_2.

The spatial frequencies of the Fourier transform are $\mu = u / \lambda D_3$ and $v = v / \lambda D_3$. Consequently changing the distance D_3 produces a scaling of the Fourier transform of the scene. The possibility of scaling the Fourier transform without changing the lens L1 inserted in the set-up gives a great flexibility to this geometry.

A second converging lens L2 with focal length F_2 produces the real image of the scene A in the correlation plane A'. A filter with a function $G*(u,v)$ is placed in the Fourier plane, being $g(x,y)$ its impulse response. Then the complex amplitude distribution $A(x',y')$ in the correlation plane is given by

$$A(x',y') = C\Psi\left(x',y',d_5 - \frac{d_5^2}{d_4+d_5-f_2}\right) \times$$

$$\iint F\left(\frac{d_3u}{\lambda},\frac{d_3v}{\lambda}\right) G*(u,v)\exp\left(i\frac{2\pi}{\lambda}\frac{d_4d_5}{d_4+d_5-f_2}(ux'+vy')\right) du\,dv \tag{2}$$

where d_i represents the inverse of the distances D_i defined in figure 1, and f_i are the inverse of the focal lengths F_i. The function Ψ is a phase factor defined as

$$\Psi(x,y,d) = \exp\left[i\frac{\pi d}{\lambda}(x^2+y^2)\right] \tag{3}$$

Except for the phase factor, $A(x',y')$ represents the Fourier transform of the product of $F(d_3u/\lambda,d_3v/\lambda)$ and $G*(u,v)$. Consequently it is the correlation of the functions $f(x,y)$ and $g(x,y)$. The correlation operation measures the similarity between the two functions $f(x,y)$ and $g(x,y)$. Correlation is one of the basic procedures in pattern recognition applications. If instead of introducing $G*(u,v)$ in the Fourier plane we consider the Fourier Transform $G(u,v)$ we are dealing with the convolution of the functions $f(x,y)$ and $g(x,y)$.
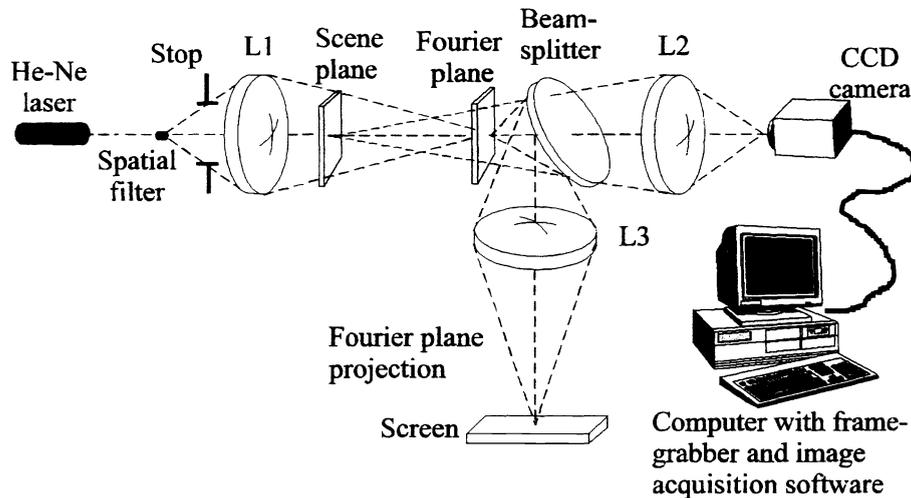

Fig.2. Experimental set-up: convergent optical correlator with two arms.

3. EXPERIMENTAL SET-UP

In figure 2 we show the scheme of the experimental set-up we use in the laboratory to teach the students the Fourier Transform properties exhibited by the optical processors and the way we can use these properties to process the optical signals. The set-up consists of a Vander Lugt converging geometry correlator, with two arms that allow the display of the Fourier plane and the final plane simultaneosly. This can be done because we insert a beam-splitter in the light wavefront

trajectory. This beam-splitter is an aluminum coated glass which is partially transmissive. By means of the lens L3 we project the Fourier plane onto the screen or onto a CCD camera. With the aid of the lens L3 we can also control the magnification of the Fourier pattern of the scene on the screen. We use the coherent light beam emitted by a He-Ne laser (λ = 632.8 nm), which is expanded and filtered with a spatial filter. Instead of using a collimated beam of light we locate the lens L1 to produce the image of the pinhole of the spatial filter at a finite distance. Lens L2 produces the Fourier Transform of the Fourier plane onto the CCD camera.

The scenes we use are recorded on photographic film. The filters we have inserted in the Fourier plane are in general binary filters but we have also made complex valued filters for optical pattern recognition tasks. These optical pattern recognition filters are computer-generated holograms[6] in which the complex values have been codified on an absorption substrate (photolite)[7] or on a phase material (bleached photographic emulsion)[8]. The focal length of the lenses L1, L2 and L3 are respectively 12 cm, 15 cm and 12 cm. We have used a monochrome 8 bits CCD camera, a Matrox Meteor-II frame-grabber and the software Intellicam provided by Matrox for image acquisition.

4. IMAGE ANALYSIS AND PROCESSING

In Section 2, we have shown mathematically the way the correlator works. Depending on the filter that we introduce in the Fourier plane we can process the input signal, that is to say the input image, for different purposes. In the next subsections we are going to show different signal analysis and processing operations that the optical correlator permits.

4.1. Frequency content

When the student gets in contact with signal processing one important point for him is to be able to interpret the frequency spectrum of a given signal. Possibly the most basic image to be understood is the diffraction grating. In figure 3(a) we show a collection of diffraction gratings, each one with a different spatial period. In figure 3(b) we show the image of the Fourier spectrum of the scene that the student can see on the screen. We perceive the corresponding diffraction orders associated with the frequency of each diffraction grating. We must say that we are dealing with binary gratings, therefore their frequency spectrum is composed of the zero order, and an infinite number of harmonics. Nevertheless, the first harmonic has notably more intensity than the others, and figure 3(b) is composed basically of the first harmonics.
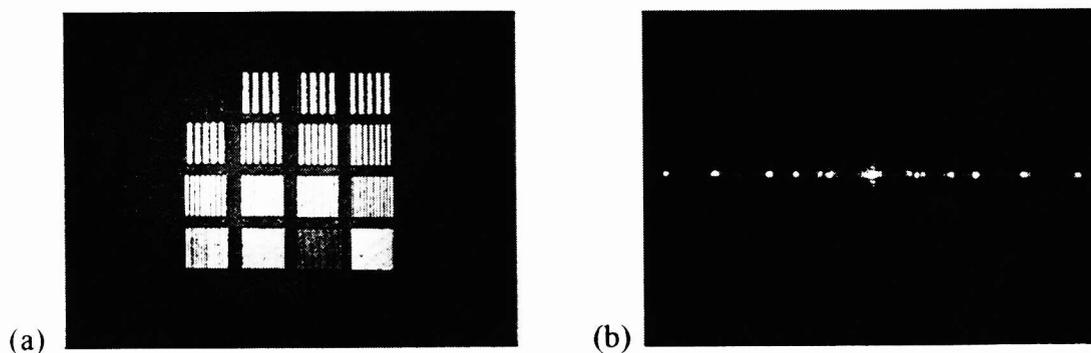

(a) (b)

Fig.3. (a) Collection of diffraction gratings with different frequencies; (b) Fourier spectrum.

The substraction of first harmonics will make apparent to which diffraction gratings they are associated with. In figure 4(a) a low pass filter has been applied. Therefore what we see is that the lowest frequency diffraction gratings are reconstructed in the correlation plane while the highest ones have disappeared. In figure 4(b) a low pass filter with a very short cuttof frequency has been applied. We can see that all the gratings have disappeared, and the only periodic feature that remains is the spacing between gratings as this corresponds to the shortest frequency in the scene. The other operation we address in

Fourier plane is high pass filtering. In figure 4(c) we see the results. Now, the gratings that remain are those with a high frequency (third row). The fourth row is not well represented even in the original image projected onto the CCD camera (figure 3).
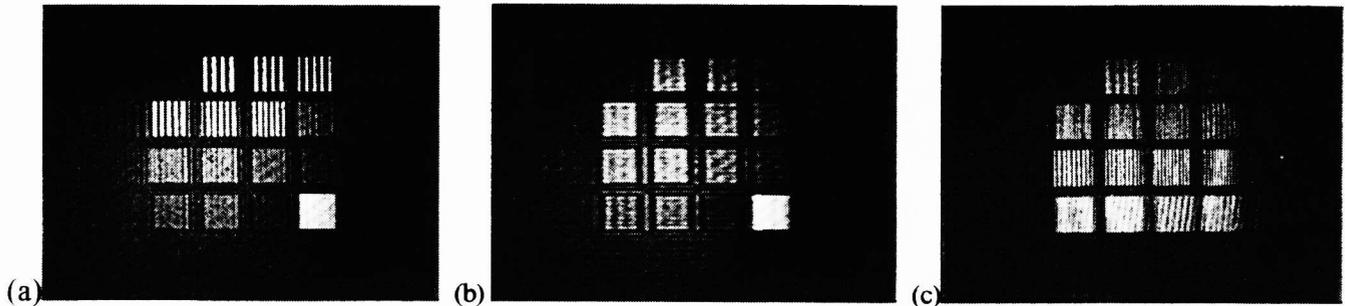

Fig.4. Reconstructed signal after application of a filter. (a) Low pass filter; (b) Low pass filter with a shorter cuttof frequency in comparison with the low pass filter (a); (c) High pass filter.

We still have another consideration to make. In figure 5 we can see the reconstruction provided when the scene is tilted with respect to figure 3(a). Now we observe clearly one diffraction grating more than in figure 3(a), where it was not well resolved. This is due to the fact that the matrix of the CCD camera is a pixelated structure, that has a limitation in the resolution of the image that can capture. When the period of the grating is shorter than the pixel spacing the grating is not well sampled and moiré effects appear. Then we can see that the cutoff frequencies of the different optical elements in the set-up play a decisive role in the different steps of image processing. In this case we can see the effect of the cutoff frequency due to the pixelated structure of the CCD camera. When the gratings are tilted the period in the horizontal direction is bigger and can be resolved by the CCD camera. Nevertheless, the two gratings in the right part of the fourth row are still not resolved by the CCD camera.
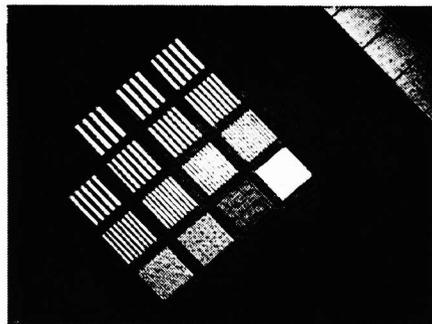

Fig.5. Tilted gratings with respect to the pixel ordering in the CCD camera.

4.2. Directional information

From the Fraunhofer diffraction pattern we can also extract information on the spatial orientation of the periodic features that appear in the scene. In figure 6(a) we show a diffraction pattern. The first harmonics are forming a circle around the zero frequency. The second harmonics are also slightly visible. In figure 6(b) we show the scene corresponding to the previous diffraction pattern. It is a collection of diffraction gratings, all of them have the same period but different orientation of the fringes.

In figure 7(a) and (b) we show the result of substracting the harmonics corresponding to specific orientations. We clearly distinguish the gratings that are removed by this filtering operation.
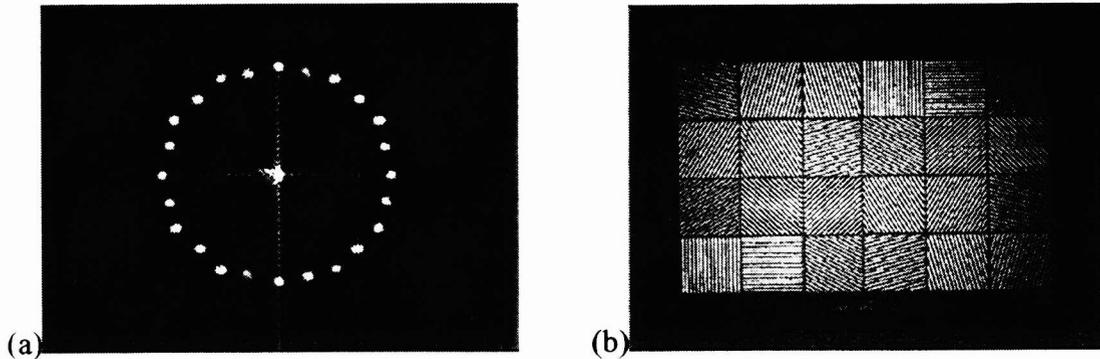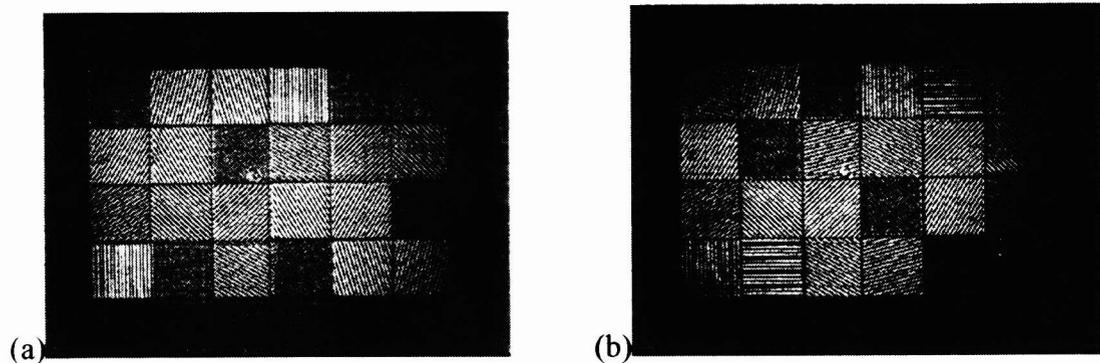

Fig.6. (a) Diffraction pattern; (b) Scene.

Fig.7. Reconstructed signal when applying orientational filters. (a) +90°, +75° and -75° orientations are removed; (b) -15° and -30° orientations are removed.

4.3. Combining frequencies and orientations

If we look at figure 8(a) we see that this diffraction pattern has one characteristic (the longitudinal disposition of diffraction orders) that resembles the previous diffraction pattern in figure 3(b) while another characteristic (the circular disposition of diffraction orders) resembles the figure 6(a). Certainly, in the scene to which this diffraction pattern corresponds (shown in figure 8(b), there exist both kinds of information, frequency and orientation. The scene is composed by four quadrants with five circles inside each one of them. The quadrants and the circles contain fringes with different periods and orientations.

In figure 9(a) we apply a low pass filter in the Fourier plane. The result is that we remove two circles in the top left quadrant and the whole top right quadrant. In figure 9(b) we show the effect of applying a directional filter that allows to pass the low frequencies. In this way there are three circles in the top right quadrant that disappear, but the rest of the periodicities in this quadrant remain unaltered, although they have the same frequency. The reason is that the orientations of the fringes are in a different direction from the ones selected by the filter.
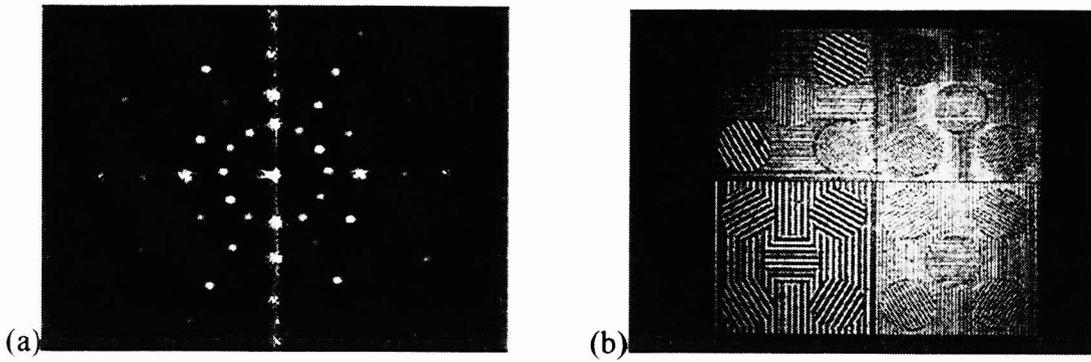
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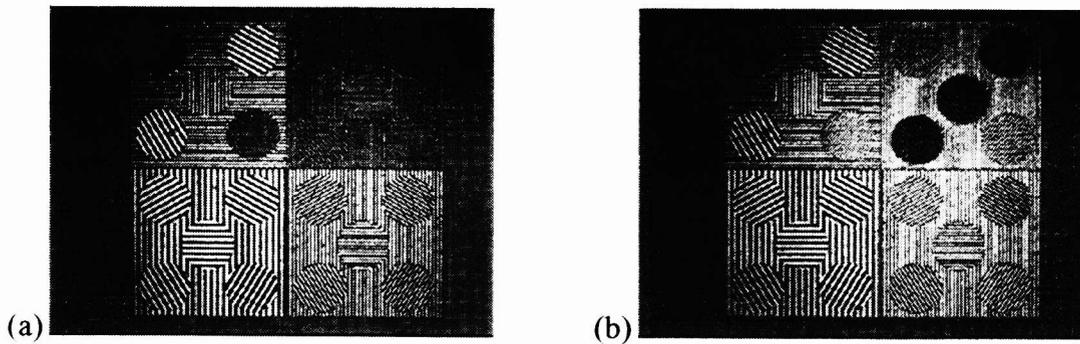Fig.8. (a) Diffraction pattern; (b) Scene.

Fig.9. Reconstructed images. (a) Low pass filter; (b) Low pass filter with orientation discrimination.

4.4. Sampling effects

Instead of geometrical images as the ones shown in the previous subsections, now we are going to examine a more realistic image. It consists, figure 10(a), in the photograph of a fireman going down a staircase. This photograph has been taken from a computer monitor. We know that computer screens are pixelated devices, then the image recorded on the photograph is in fact a sampled image. In figure 10(b) we show the diffraction pattern of the scene. The hexagonal periodicity in the diffraction orders is due to the hexagonal distribution of the pixels in the monitor we used.
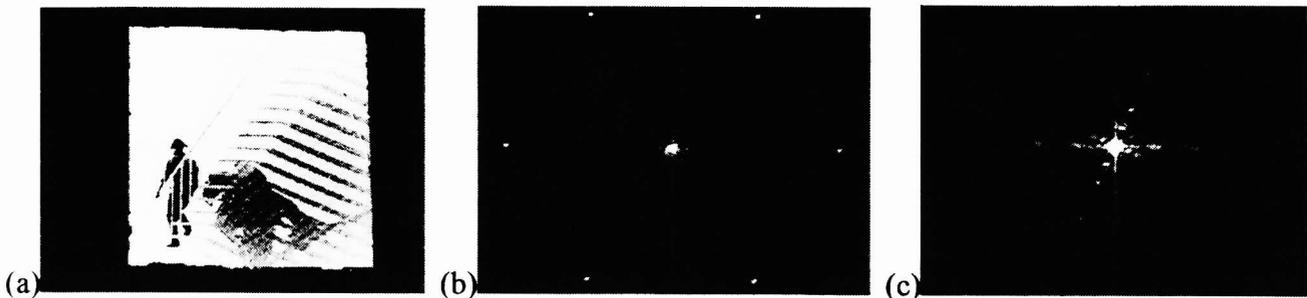

Fig.10. (a) Scene; (b) Diffraction pattern; (c) Magnified diffraction pattern.

When we make the Fourier Transform of a sampled image the result we obtain is the convolution of the Fourier Transform of the image by the Fourier Transform of the sampling function[1]. Then, in each one of the diffraction orders of the hexagonal pattern shown in figure 10(b) we have the Fourier Transform of the image. If we examine the zero frequency of the hexagonal pattern with a higher magnification, figure 10(c), we find that this zero frequency has an internal structure. We appreciate diffraction orders along three different directions (vertical, horizontal, and at approximately 30°). This is precisely the Fourier Transform of the image. In the scene, figure 10(a), we find the three periodicities: spacing between the steps of the staircase, spacing between the shadow of the steps, and spacing between the bars of the handrail.
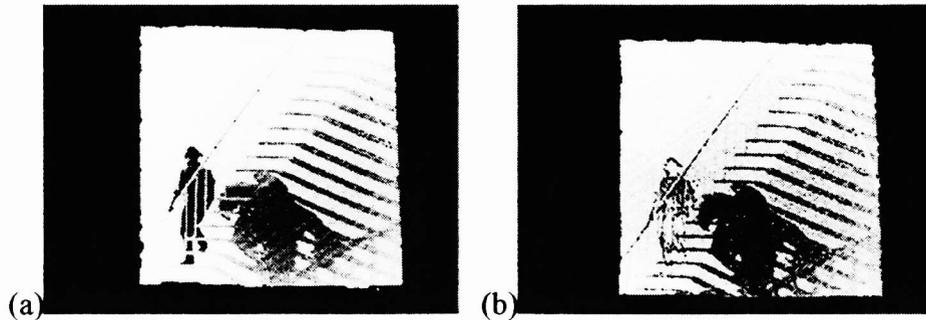

Fig.11. Reconstructed signal when only the zero frequency of the hexagonal diffraction pattern passes (a), and when only one of the harmonics of the hexagonal diffraction pattern passes (b).

From the sampling theorem we know that if the sampling frequency is high enough we can recover the original continuous signal from its discrete counterpart with no loss of information[4]. In the Fourier plane we have applied two different filters. On one side we have applied a blocking filter that blocks all the Fourier spectrum with the exception of the zero frequency of the hexagonal pattern. The reconstructed signal is shown in figure 11(a). On the other side we have applied a blocking filter that blocks all the Fourier spectrum with the exception of one of the harmonics of the hexagonal pattern. The reconstructed signal is shown in figure 11(b). We can see in figure 11(a) and in figure 11(b) that we recover another time the original input scene, although in figure 11(b) it is recovered with more noise.

5. OPTICAL PATTERN RECOGNITION

In Optical Pattern Recognition the goal is to detect and to determine the position of an object inside an input scene. The optical correlator has been widely used in optical pattern recognition. In Section 2 we told that the correlation provides a measure of the similarity between two functions. In this case the filter that is used in the Fourier plane is complex valued and it is related to the Fourier Transform of the target object to be detected. The complex transparency can be produced by holographic interference, as in the case of the Vander Lugt filter[5], or it can also be produced by digital methods like a computer-generated hologram.

The phase of a complex field is a magnitude that can not be directly visualized because the detectors simply measure intensity. It is in this sense that the application of the optical correlator to pattern recognition demmands a larger maturity of the student in signal processing concepts, as now we are dealing with a filtering operation that not only modifies the intensity of the Fourier Spectrum of the scene but also alteres its phase. Then we think that this is an application that can serve the student to have an idea of the large number of possibilities that Signal Processing offers to him.

The filters we use are designed by digital methods and they are recorded by means of a resolution graphic device on a photolite, which is an absorption substrate. We do also have correlation filters made on bleached photographic emulsion, which is a phase material. In figure 12 we show a magnified image of this filter made on a photolite. The complex

amplitude information of the Fourier Transform of the object to be detected is codified by means of the position and the area of rectangular apertures. The codification method that we follow was proposed by Burckhardt[9]. In figure 13 we show the scene we work with. It consists of two butterflies, and the one we want to recognize is butterfly (B).
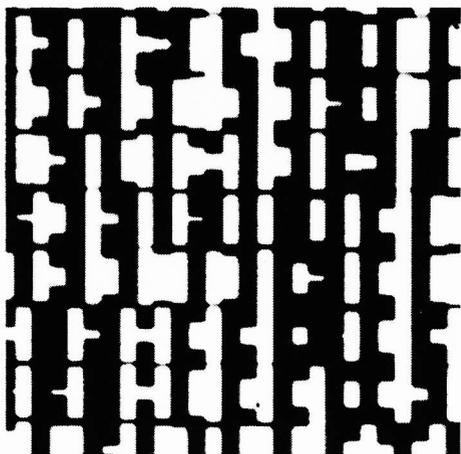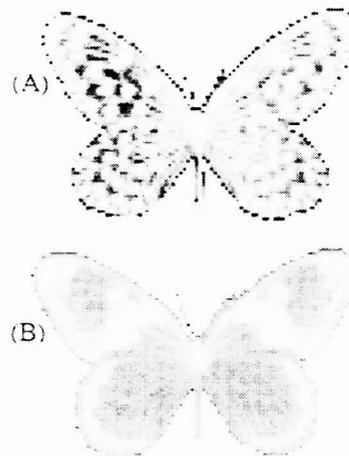

Fig.12. Magnified image of the correlation filter.

Fig.13. Scene.

If we put the filter in the scene plane and we look at its Fourier Transform we obtain the image presented in figure 14. We can see that we recover the butterfly with the edges enhanced. In fact in the digital design of the filter we have simply considered the information about the phase distribution in the Fourier Transform of butterfly (B), this is what we call a phase-only filter. It results in an enhancement of the higher frequencies, and it emphasizes the edges of the butterfly. We can also see that the butterfly is repeated with a squared periodicity. This is a consequence that the information contained in the computer-generated hologram is sampled with this squared periodicity.
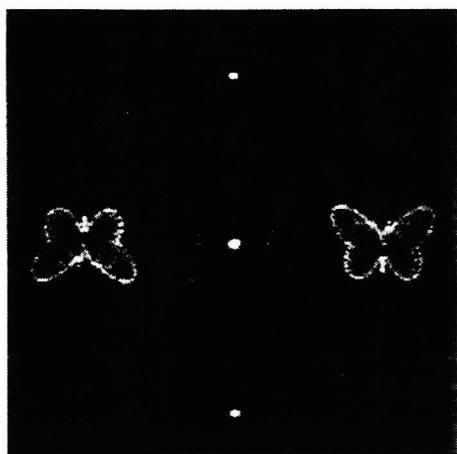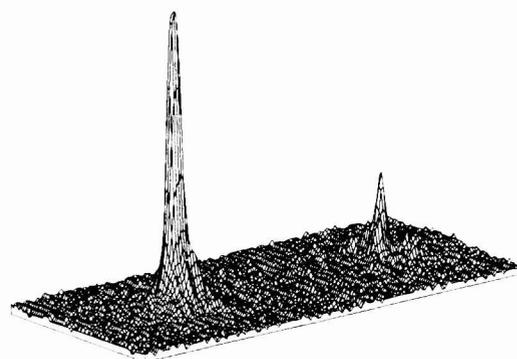

Fig.14. Impulse response of the correlation filter.

Fig.15. 3-D representation of the correlation plane.

In figure 15 we have represented the result obtained from the correlation operation using the scene and the filter described previously. It is a three dimensional representation of the correlation plane. The higher peak indicates the location of butterfly (B) inside the scene. Then, butterfly (B) have been clearly discriminated from butterfly (A).

6. SUMMARY

In the present discussion we have presented the possibilities that an Optical Correlator set-up offers as a tool for students training in Signal Processing. The processing of both optical and electrical signals follows the same theoretical principles, and therefore an Optical Correlator can be useful for teaching these principles not only for people working on Optics but also for electrical engineers and people working on other signal processing areas. We have mainly concentrated on the importance of understanding the Fourier spectrum of a signal, and we have given different examples of filtering operations that can be applied to the Fourier spectrum. Our Optical Correlator set-up with two arms enables the display of the Fourier plane and the final plane simultaneously. As a more sofisticated signal filtering operation we have introduced the application of the optical correlator to optical pattern recognition.

ACKNOWLEDGMENTS

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