On the Properties of Optical Waves in Turbid Media

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ABSTRACT

Optical measurements represent a valuable tool for in vivo analysis of tissue properties, such for example the average level of oxygenation of perfusing blood. A general problem in turbid medium, such as in the case of most tissues, is to distinguish between phenomena caused by absorption and those due to scattering. These problems can be overcome by using either time or frequency domain techniques. Frequency domain measurements are based on evaluation of the phase and amplitude information of transmitted amplitude modulated optical beams. These type of measurements might prove to be a valuable tool for in situ evaluation of tissue properties such as for example fluorescence and absorption. Irradiation of turbid media by harmonically modulated optical beams will initiate density waves of diffusely propagating photons. The phase velocity of these waves will be quite different from the velocity of light. The velocity, which is strongly dependent on the modulation frequency, can typically vary from the velocity of light as an upper limit and down to about 10% of this value in highly scattering, moderately absorbing tissues. This presentation will give a brief discussion of the general properties of this kind of waves in tissues.

INTRODUCTION

Biological media have, because of is complicated structure, very complex optical properties. Tissues are from an optical point of view very irregular, inhomogeneous media. The optical inhomogeneities are due to differences in optical polarizability as well as in the optical absorption properties. These differences exist between the cells and their surroundings as well as within each individual cell.

The variations in the polarizability can be characterized in terms of the index of refraction. This quantity, which expresses the ratio between the velocity of light in vacuum to that of the pertinent medium, varies from that of pure water, i.e. $n = 1.33$, as a lower limit and up to about $n = 1.5$. The spatial fluctuations in the refractive index initiate refraction and reflections of the optical waves. A gradual transversal variation of the index of refraction across the optical wavefront will initiate a curved path of the beam; the trajectory will bend towards the region with the higher index of refraction. A variation in the index of refraction of about 10 % over a typical cellular dimension of about of 10 μm will result in a radius of curvature of about 0.1 mm.

The propagation of light in tissue is strongly influenced by this mechanism; light is very efficiently scattered out of any incident collimated beam.[1)] The light will typically be scattered into an almost isotropic distribution within a few millimeters from the source. In some heavy scattering tissues, such as in the case of adult brain tissues, this randomization takes place within a fraction of a millimeter. The only important exception from this rule is ocular tissues, where an image is formed on the retina after propagating about 25 mm through the cornea, the lens and the vitreous.

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The total optical radiation in tissues is therefore composed of waves arriving from different directions with different optical phases. An important question is whether the phase relationships between these various waves are slowly varying or if they vary in a rapid and stochastic manner. The waves subjected to the first condition will add in a partially coherent manner. The total optical power will then be the determined by the average value of the square of the total electric field. The power might then be higher or lower than the sum of the optical power in each wave, dependent on whether the waves add constructively or destructively. The waves with rapid and stochastic phase fluctuations will, on the other hand, add incoherently. The total optical power will then always be the sum of the power of all participating waves.

The incoherently added optical power can be characterized by quantities defined as the radiance and the radiant energy fluence rate. The radiance is defined as the optical energy flux in some direction per unit solid angle and per unit area orthogonal to this direction. The radiant energy fluence rate is defined as the optical energy flux incident on an infinitesimal small sphere divided by the cross-sectional area of that sphere. The radiant energy fluence rate, which is a measure of the total optical flux, is thereby defined as the integral of the radiance over all directions.[2] The relation between the radiance and the radiant energy fluence are thus defined by,

$$\varphi = \int_{\Omega=0}^{4\pi} L \, d\Omega \qquad (1)$$

where L and φ are, respectively, the radiance and the radiant energy fluence rate. The element of solid angle is given by $d\Omega$, and the integration is taken over all spatial direction, i.e. over a solid angle equal to 4π.

The radiance in a completely isotropic light field is, as follows from this equation, given by $L = \varphi/4\pi$. The corresponding irradiance E, which is defined as the radiant energy flux on an element of surface divided by the area of that element, can be expressed,

$$E = \int_{\Omega=0}^{2\pi} L(-\vec{l}\cdot\vec{n}) d\Omega = \pi L = \frac{\varphi}{4} \qquad (2)$$

where $\vec{n}$ and $\vec{l}$ are, respectively, the outward unit surface normal and the unit vector along the axis of the element of solid angle. The integration is taken over half space, i.e. over a solid angle equal to 2π. This integral is illustrated in fig.1. The optical flux $Ld\Omega$, which is incident along the axis of the solid angle $d\Omega$, gives the flux per unit area orthogonal to this axis. The projection of the unit surface element with outward unit normal $\vec{l}$ onto this axis is given by $-\vec{l}\cdot\vec{n}$.

The optical energy will arrive at a distal location in the tissue through a mechanism very much of the same nature as an ordinary diffusion process; at any location distal to the irradiated surface any incident collimated optical beam will have been scattered into diffusely propagating photons. The radiance can, however, not be completely isotropic; any net transport of diffuse photons in some direction must be expressed by a higher radiance in that direction. The radiance should therefore be expressed by a series expansion where the expansion parameter is the deviation from the isotropic distribution.

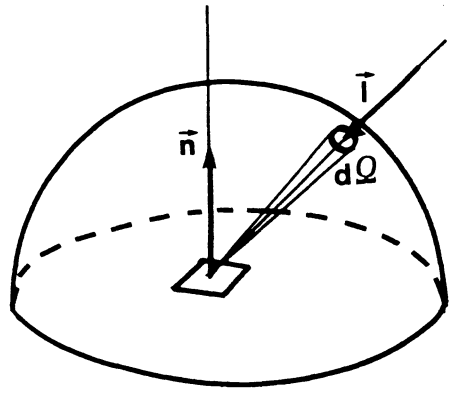

Fig.1 Irradiation of an element of surface

The irradiation onto an element of surface will vary with its orientation with respect to the direction of the net flux of diffuse photons. The maximum and minimum value which will be obtained, respectively, when the surface normal is antiparallell or parallel to this direction can be expressed,

$$E = \frac{\varphi}{4} \pm \frac{j}{2} \qquad (3)$$

where j is the magnitude of the transport vector. This transport vector $\vec{j}$ represents a collimated flux of diffuse photons. The irradiation is thus enhanced by the quantity j/2 on the side facing the flux and correspondingly decreased by the same quantity on the back side; the total deviation from the completely isotropic case equals j.
The radiance should then be expressed by a series expansion of the form,

$$L = \frac{\varphi}{4\pi} + \alpha \vec{j} \cdot \vec{l} + \cdots$$

(4)

where α is the constant. This constant can easily be determined from the condition given by eq.3,

$$E = \int_{\Omega=0}^{2\pi} [\frac{\varphi}{4\pi} + \alpha(\vec{j} \cdot \vec{l})(-\vec{n} \cdot \vec{l})] d\Omega = \frac{\varphi}{4} + \frac{2\pi\alpha}{3} j$$

(5)

When the value $\alpha = 3/4\pi$ is substituted in eq.4. the radiance can be expressed,[3)]

$$L = \frac{\varphi}{4\pi} + \frac{3}{4\pi} \vec{j} \cdot \vec{l} + \cdots$$

(6)

where the first term on the right hand side corresponds to the completely isotropic case and the second term represents the net transport of diffuse photons.

The net transport of these diffuse photons, which in an heuristic description will occur from regions with high fluence rate to regions with smaller values, can be expressed in the form,

$$\vec{j} = -\zeta \, grad\varphi$$

(7)

where ζ is a diffusion constant for diffusely propagating photons. This diffusion constant can be expressed in term of the scattering and absorption properties. This diffusion constant, which is equal to the reciprocal transport cross section σ_{tr}, can be expressed,[3)]

$$\zeta = \frac{1}{\sigma_{tr}} = \frac{1}{3(\sigma(1-g)+\beta)} = \frac{1}{3(\sigma_{eff}+\beta)}$$

(8)

where σ, g and β are, respectively, the scattering coefficient, the average cosine of the scattering angle, and the absorption coefficient. The effective scattering coefficient σ_{eff} is here a priori much larger than the absorption coefficient.

The net flux of diffuse photons propagating out of a unit volume , which is characterized by the quantity $div\,\vec{j}$, can be expressed,

$$div\vec{j} = -\frac{1}{c}\frac{\partial\varphi}{\partial t} - \beta\varphi + q$$

(9)

where c is the velocity of light in tissue. The first term on the right hand side represents the rate of change in the energy density φ/c , and the second term characterizes the power density $\beta\varphi$ lost by absorption. The third term gives the source density q which represents the rate of generation of diffuse photons per unit volume. The generation of diffuse photons may for example be generated through scattering from an incident collimated beam or by mechanisms such as fluorescence or chemiluminescence.

These equations may be combined in the form, eq.7 and 9 [4)]

$$X\nabla^2\varphi - \frac{\partial\varphi}{\partial t} - \frac{\varphi}{\tau} = -qc$$

(10)

where X is the optical diffusivity and τ is the optical absorption relaxation time. The parameters are defined by,

$$X = c\zeta \; ; \quad \tau = \frac{1}{c\beta}$$

(11)

The one-dimensional solutions of this equation may be expressed in terms of plane harmonic waves the form,

$$\varphi = \varphi_0 e^{i\omega t - kz}$$

(12)

where ω and k are, respectively, the angular frequency and the complex angular wavenumber.

The solution written in terms of the real and imaginary part of the wavenumber yield,

$$\varphi = \varphi_0 e^{i(\omega t - k_i z)} e^{-k_r z}$$

(13)

where k_i and k_r are, respectively, the real and the imaginary part of k. The imaginary part of the k expresses the angular phase-shift per unit length whereas the real part characterizes the attenuation. The wavelength λ and the phase velocity v_{ph} are given by,

$$\lambda = \frac{2\pi}{k_i} \; ; \; v_{ph} = \frac{\omega}{k_i}$$

(14)

It is, however, important to note that this wavelength and phase velocity are not the velocity or the wavelength of the individual photons. The parameters describe, respectively, the minimum distance in space between regions with the same phase of diffuse photon density and the corresponding velocity of propagation of the phase front. The parameters thus express the properties of the collective diffuse photon density rather than the properties of the individual photons.

The situation is somewhat analogous to heat transport. The wavelength and phase velocity of thermal waves do not express the properties of the individual phonons or electrons; these parameters express the property of the temperature which again is a measure of the density of diffusely propagating phonons or electrons.

The complex wavenumber can according to eq.10 , be expressed,[5]

$$k_r = \frac{1}{\sqrt{2X\tau}} \sqrt{\sqrt{1+(\omega\tau)^2}+1}$$

$$k_i = \frac{1}{\sqrt{2X\tau}} \sqrt{\sqrt{1+(\omega\tau)^2}-1}$$

(15)

The waves have very different characteristics in the various frequency regions.

In the high frequency region, i.e. $\omega >> \tau^{-1} = c\beta$, the real and the imaginary part of the wavenumber are equal. This value is,

$$k_i = k_r = \sqrt{\frac{\omega}{2X}}$$

(16)

These properties imply that the waves are heavily attenuated with a constant attenuation equal to 2π per wavelength. The amplitude is therefore reduced by 27 dB, i.e. to about 0.2% of the initial value over a distance corresponding to one wavelength.

The waves also exhibit a frequency dependent phase velocity. This velocity is given by,

$$v_{ph} = \frac{\omega}{k_i} = \sqrt{2X\omega}$$

(17)

This phase velocity decreases with the reduction in frequency of the periodic variations of the photon density. The reason for this phenomenon is that a change in the local fluence rate is made up by photons collected from a surrounding region; and this region increases in size as the frequency goes down i.e. when the energy from larger distances is allowed to be collected.

By the same argument the phase velocity will increase with increasing frequency. The photons are then only collected from more proximal regions and the time required to constitute the steady state distribution is consequently reduced. However, the reduction in the region of collection also reduces the number of

photons received. The attenuation is therefore increased with frequency in such a manner that the attenuation per wavelength remains constant. There will, of course, be an upper frequency limit for the validity of eqs.16 and 17. The upper frequency must, since the validity of the diffusion theory is based on multiple scattering, be lower than the reciprocal average time between two scattering events. The upper frequency is therefore determined by,

$$\omega \ll c\sigma_{eff} \sim \frac{c}{\zeta} = \frac{c^2}{X} \tag{18}$$

An order of magnitude estimate for the upper frequency limit follows from the linear dimensions and the scattering properties of the cells. In tissues with typical cellular dimensions of 10 μm, average cosine of the scattering angle of the order of 0.9, and velocity of light of about 2.10^8 m/s, the effective scattering coefficient will be in the range of σ_{eff}= 100 cm^{-1}. The upper frequency region will be of about 300 GHz, corresponding to a time scale of about 0.5 ps.

The wave properties are significantly different in the low frequency region. These properties are here rather dominated by the absorption than by the scattering. The attenuation and the phase velocity can be expressed,

$$k_r = \frac{1}{\sqrt{\tau X}}$$

$$v_{ph} = \frac{\omega}{k_i} = 2\sqrt{\frac{X}{\tau}} \tag{19}$$

The effect of the absorption process is an enhanced attenuation together with an increased phase velocity. The reasons for these phenomena are simply that the absorption now limits the extent of the region where the photons are collected. The attenuation increases because the number of photons, which make up the local fluence rate, is reduced. The phase velocity is increased because the time required to collect these photons is reduced.

The frequency dependence of attenuation and phase velocity in the low frequency region are therefore significantly different from the high frequency region. In fact, as can be seen from eq.19 , the attenuation and phase velocity are both frequency independent in the low frequency region.

An order of magnitude estimate for the frequency limits of this region follows from the absorption coefficient. Typical values for near infrared optical absorption coefficients for non-pigmented tissues are in the range of β= 0.2- 1 cm^{-1}. The frequency limit is therefore in the range of 0.5- 3 GHz corresponding to a time scale of about 0.1 ns.

This frequency limit increases with increasing attenuation. But, since the validity of the diffusion theory requires absorption coefficient always is smaller than the scattering coefficient, the upper frequency must at least be about one order of magnitude below the frequency limit given by the scattering properties, i.e. eq.18.

If, on the other hand, the absorption coefficient becomes significantly larger than the scattering coefficient the phase velocity will approach the velocity of light in the medium.

The frequency dependence of the real and the imaginary part of the wave number is demonstrated in fig.2. The optical diffusivity and the absorption relaxation time are, respectively, taken X= 20 000 m^2/s and τ= 50 ps. These values correspond to typical parameters for tissues with high scattering and moderate absorption; the corresponding values for the absorption coefficient and the transport cross

section are, respectively, $\beta = 1\ cm^{-1}$ and $\sigma_{tr} = 100\ cm^{-1}$.

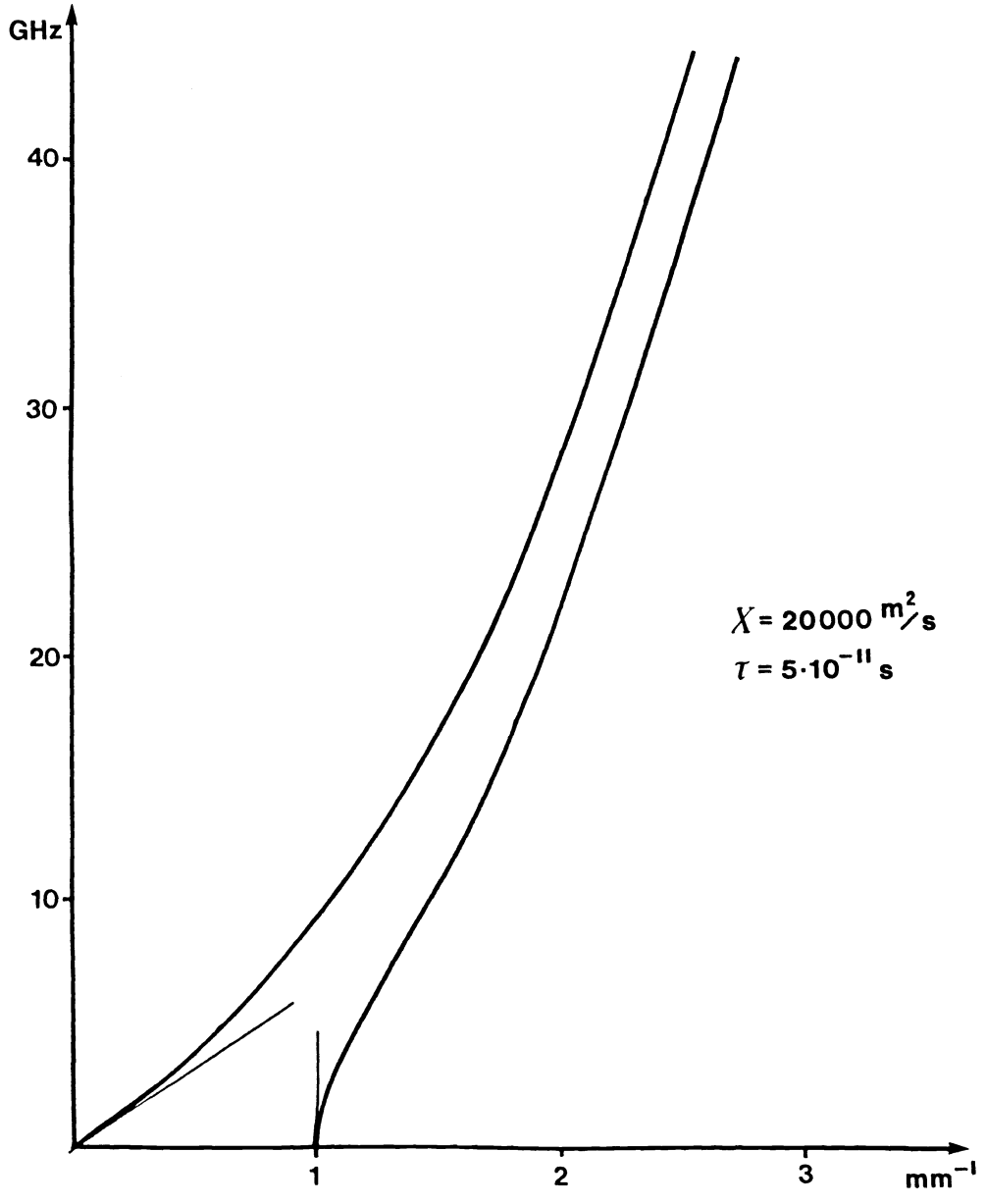

Fig.2. Dispersion relation between frequency and angular wavenumber. Upper curve gives relation between frequency in GHz and angular wavenumber in mm^{-1}. Lower curve gives relation between frequency in GHz and attenuation coefficient in mm^{-1}.

The upper and the lower curves in fig.2 express, respectively, k_i and k_r as given in eq.15. The figure demonstrates that the phase velocity, which is given by the tangent to this curve, increases with increasing frequency. The high frequency region exhibits strong dispersion dominated by the diffusivity (eq.17) whereas in the low frequency limit the phase velocity reaches a dispersion-less lower limit, $v_{ph} = 0.2$ c, as given by eq.19. The attenuation coefficient which is given by k_r, decreases with frequency and reaches a lower value of $k_r = 1\ mm^{-1}$ for frequencies well below the reciprocal absorption relaxation time, i.e. below $f = 1/2\pi\tau = 3.2$ GHz.

The same results are presented in a somewhat different form in fig.3. The upper curve shows here the frequency dependence of the optical penetration depth, $\delta = 1/k_r$, relative to the low frequency value of $\delta = 1$ mm, and the lower curve gives the corresponding dependence of the ratio between the phase velocity and the velocity of light in tissue. The phase velocity varies as follows from the figure, from 20% and up to about 50% of the velocity of light over the frequency region from zero and up to about 40 GHz. The entire diffusion theory approach becomes invalid for frequencies above $f = c^2/2\pi X = 300$ GHz where the dynamics of the individual photons has to be considered.

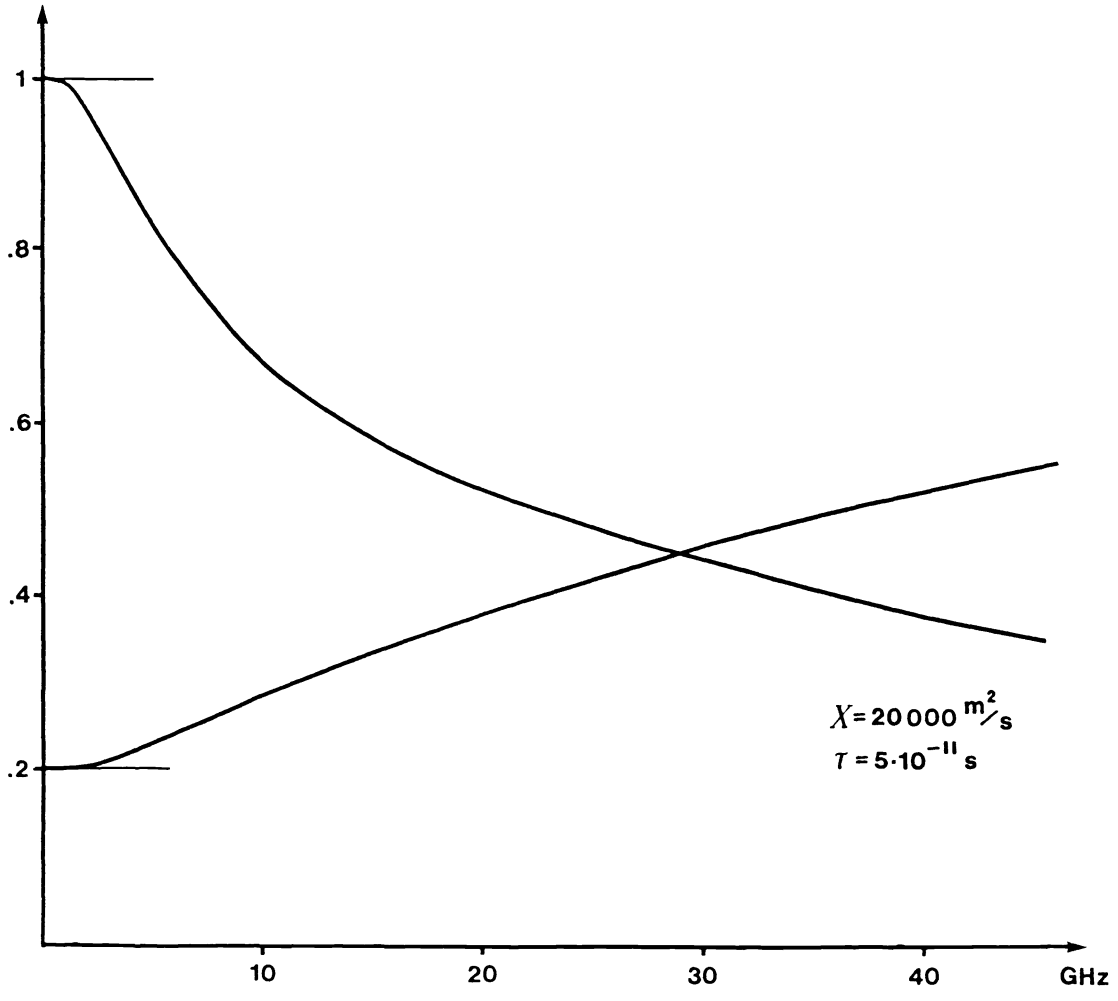

Fig.3. Optical penetration depth and phase velocity versus frequency. Upper curve gives the penetration depth relative to the penetration depth at zero frequency. The lower curves gives the phase velocity relative to the velocity of light.

The time dependent optical distribution from a localized optical point source generating a constant amount of diffuse optical power from time t=0 can be expressed,

$$\varphi = cP_0 \int_0^t \frac{1}{8\sqrt{(\pi X(t-\xi))^3}} e^{-\frac{r^2}{4X(t-\xi)}} e^{-\frac{(t-\xi)}{\tau}} d\xi$$

(20)

where P_0 is the constant optical power and r is the distance from the source.

The corresponding steady state distribution becomes,

$$\varphi = \frac{cP_0}{4\pi Xr} e^{-\frac{r}{\sqrt{\tau X}}}$$

(21)

The steady state distribution for the harmonically modulated source can be expressed,

$$\varphi = \frac{c}{4\pi Xr}[P_0 e^{\frac{-r}{\sqrt{\tau X}}} + P_\omega e^{-\frac{r}{\sqrt{2\tau X}}\sqrt{\sqrt{1+(\omega\tau)^2}+1}} \cos(\omega t - \frac{r}{\sqrt{2\tau X}}\sqrt{\sqrt{1+(\omega\tau)^2}-1})]$$

(22)

where P_0 and P_ω are, respectively, the average value and the modulation amplitude of the source power.

The optical diffusivity and the relaxation time can, as follows from this expression, be determined experimentally by measurements of the phase and attenuation of the photon density waves. The relations between the diffusivity and relaxation time and the wave properties follow from eq.15,

$$X = \frac{\omega}{k_r k_i}$$

$$\tau = \frac{2k_r k_i}{\omega(k_r^2 - k_i^2)}$$

(23)

Accurate determination of the attenuation will require measurements in the low frequency region ($\omega << 1/c\beta$) where the attenuation has a significant impact on the properties of the waves. Small inaccuracies in the measured values for k_i and k_r in high frequency region will, as follows from eq.16, result in large uncertainties in the absorption coefficient.

The diffusivity will, on the other hand, be determined with optimal accuracy in the high frequency region ($1/c\beta << \omega << 1/c\sigma_{tr}$) where the scattering has a predominant influence on the wave properties.

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