

Step-function optical distributions
in cross-sectional shifts measuring

Yuri Kirchin

St.Petersburg Institute of Fine Mechanics & Optics
Department of Opto-Electronics. St.Petersburg USSR

The main principles of opto-electronic devices for fine cross-sectional shifts measuring based on the step-function (SF) optical distributions are discussed. Basic mathematical method and its practical development are suggested.

If in the subject space of non-ideal lens optical energy is distributed as a composition of two SFs of X -coordinate normal to the optical axis

$$\Phi_1(X) = \Phi_0 \sigma(X); \quad \Phi_2(X) = \Phi_0 \sigma(-X)$$

the resulting lightness distribution E in the image space will have two components $E_1(X)$ and $E_2(X)$ with the crossing lying on the optical axis on every distance. The crossing estimation may be used as a metrological base for the CCD-sensor shifts measuring. The shift value is to be found in the equation

$$E_1(X) = E_2(X) \quad (1)$$

where X - the CCD coordinate is to be transformed to the space one X . Every CCD line transforms continuous distributions E_1 and E_2 to vectors

$$\mathbf{E}_1 = \{E_{1j}\}_{j=1}^n; \quad \mathbf{E}_2 = \{E_{2j}\}_{j=1}^n$$

stochastically depending upon the sensor coordinate X . Continuous approximating functions $f_1(X)$ and $f_2(X)$ are to be used for fine crossing estimation

$$f_1(X) = f_2(X) \quad (2)$$

instead of (1). Finding of the f -functions according to the least squares method (LSM) is suggested in the form of the approximating functions' space basis ϕ_ν line combination

$$f(X) = \sum_{\nu=0}^N a_\nu \phi_\nu(X) = \mathbf{F}\mathbf{A}.$$

Line model of the observation structure is $\mathbf{E} = \mathbf{F}\mathbf{A}$ and LSM-estimation of the coefficients' vector \mathbf{A} and f -functions are

$$\hat{\mathbf{A}} = (\mathbf{F}^T \mathbf{F})^{-1} \mathbf{F}^T \mathbf{E}; \quad \hat{f}_1 = \mathbf{F}\hat{\mathbf{A}}_1; \quad \hat{f}_2 = \mathbf{F}\hat{\mathbf{A}}_2.$$

They are to be used in (2). Interval estimation is possible.¹ For the CCD-defects' signals and noise filtration the finite differences method (FDM) is useful. Finite differences are defined

$$\Delta E_j = E_{j+1} - E_j; \quad \Delta^2 E_j = \Delta E_{j+1} - \Delta E_j.$$

If additional error component e takes place on the j -pixel its value may be estimated as

$$\hat{e}_j = -0.5 \Delta^2 E_{j-1}.$$

Co-application of the SF-distribution, FDM and LSM allows to get shift's estimation with methodical error less than 0.01 CCD pixel size.

1. Ю.Г.Киричн, "Применение приборов с зарядовой связью для определения положения оптической равносигнальной зоны", *Известия вузов СССР.-Приборостроение*, N 7, pp. 12-15, 1991.