TUNNELING OF ULTRASHORT EM WAVE PULSES IN GRADIENT METAMATERIALS: PARADOXES AND PERSPECTIVES.

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Abstract

Amplitude – phase spectra of IR waves tunneling through a gradient dielectric nanophotonic barrier, found in the framework of an exactly solvable model of this medium, are used for optimization of superluminal reshaping of tunneling pulses. This barrier, characterized by a cut-off frequency Ω, determined by the shape of distribution of refractive index across the barrier, provides a tunneling regime for waves whose frequencies are less than Ω. In a spectral range located nearby this cut-off frequency Ω, an almost reflectionless tunneling of these waves occurs, accompanied by large strongly dispersive phase shifts. These shifts outstrip in some spectral range the phase shifts accumulated by the same harmonics along the same way in free space. Depending on the detuning between the pulse carrier frequency ω_0 and Ω the interplay between superluminal (tunneling) and subluminal (transparent) harmonics results in an ultrafast reshaping of the transmitted waveform, yielding a pulse spatial broadening, formation of superluminal precursors at the front edge of transmitted pulse and the splitting of pulse's maximum, while the displacement of the centre of gravity of reshaped pulse as well as the velocity of energy transfer stay subluminal.

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1. Introduction.

Tunneling is a fundamental phenomenon in the dynamics of waves of various physical nature. The interest in it was aroused after Gamov's famous work (1928) on nuclear alpha decay [1]. Three years later Condon's calculation [2] of the velocity or transit time of a tunneling particle, attempted in the framework of this new theory, revealed a basic problem: how to define these quantities in the "classically forbidden" zone, where the particle momentum should be assigned imaginary values. A year later MacColl [3] came to the conclusion that "there is no appreciable delay in the transmission of the wave packet through the barrier", which seemed to be an indication of a superluminal velocity of the tunneling particle, in contradiction with the Einsteinian limit. Later on the advent of lasers and the formal similarity of stationary Schrodinger equation and Helmholtz wave equation gave rise to a series of attempts of optical measurements of the velocity of tunneling electromagnetic waves by means of different versions of FTIR (frustrated total internal reflection) devices. The experiments with bi-prism devices [4] as well as the analysis of the spatial displacement of the peak of a tunneling pulse [5] and the analysis of photon's tunneling time [6-8] were considered by some authors to favor the concept of superluminal phase time for the tunneling EM waves [9]. However, this concept aroused conflicting viewpoints [10 – 11].

During the last decades a growing attention to this problem leads to a series of theoretical works, based on such new concepts as negative [12] and complex [13] phase time or on a reformulated version of the causality principle [14]. These works were treating the fundamental problem: is the Einsteinian limit valid for tunneling EM waves too?

2. Amplitude-phase spectra of waves traversing the gradient nanolayer: traveling and tunneling regimes.

We consider a linearly-polarized EM wave with components E_x and H_y, propagating in the z-direction, impinging perpendicularly on the interface of a thin heterogeneous dielectric film (gradient wave barrier). Supposing the film to be lossless and non-magnetic, and expressing the field components through the vector-potential $\boldsymbol{A}$ ($A_x = \psi$, $A_y = A_z = 0$) so that :

$$E_x = -\frac{1}{c}\frac{\partial \psi}{\partial t}, \quad H_y = \frac{\partial \psi}{\partial z} \tag{1}$$

one can reduce the system of Maxwell's equations related to this geometry to one equation, governing the function ψ in the layer:

$$\frac{\partial^2\psi}{\partial z^2} - \frac{\varepsilon(z)}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0 \tag{2}$$

Here $\varepsilon(z)$ is the dielectric susceptibility, characterized by the coordinate-dependent profile $\varepsilon(z) = n_0^2 U^2(z)$, where n_0 is the value of refractive index on the interface of the gradient barrier, and the function $U(z)$ represents the profile of refractive index inside the barrier. We will consider below the symmetric concave profile (Fig.1):

$$U(z) = \left(1 + \frac{z}{L_1} - \frac{z^2}{L_2^2}\right)^{-1} \tag{3}$$

More general profiles, both concave and convex can be treated as follows [15], but we restrict ourselves here to profile (3) which possesses the specific properties that we intend to discuss here. Spatial scales L_1 and L_2 are connected with the film's thickness d and minimal value of ε ($\varepsilon_m = n_0^2 U_m^2$) by :

$$L_1 = \frac{d}{4y^2}; \quad L_2 = \frac{d}{2y}; \quad y = \sqrt{U_m^{-1} - 1} \tag{4}$$

The nanolayer is located on a thick homogeneous substrate with refractive index n. The values of d and U_m in this model remain arbitrary. Transforming eq (2) in a new coordinate system attached with the optical path in the nanolayer

$$F = \psi\sqrt{U(z)} \quad ; \quad \eta = \int_0^z U(z_1)\,dz_1 \tag{5}$$

allows to rewrite the propagation equation (2) under the form :

$$\frac{d^2F}{d\eta^2} - F\left[\left(\frac{\omega n_0}{2}\right)^2 - p^2\right] = 0 \quad ; \quad p^2 = \frac{1}{4L_1^2} + \frac{1}{L_2^2} \tag{6}$$

Solution of (6) for the gradient film takes the form of regular harmonic forward and backward waves with wavenumber q, travelling along the η – axis:

$$F = A\left[\exp(iq\eta) + Q\exp(-iq\eta)\right]\exp(-i\omega t) \tag{7}$$

Here A is some normalization constant, and the dimensionless factor Q represents the contribution of the backward wave to the field inside the film, quantities q and N are:

$$q = \frac{\omega n_0 N}{c} \quad ; \quad N = \sqrt{1 - \frac{p^2 c^2}{\omega^2}} \tag{8}$$

The solution of wave equation (2), given by function ψ, now writes:

$$\psi = \frac{A\left[\exp(iq\eta) + Q\exp(-iq\eta)\right]\exp(-i\omega t)}{\sqrt{U(z)}} \tag{9}$$

Substitution of function ϕ (9) into equations (1) brings the electric and magnetic components of EM field at any point inside the film. As shown by eq (8), the profile (3) possesses a plasma-like, heterogeneity-induced dispersion (*N*, through *p*, depends on the geometrical characteristics of the index profile only) with a cut-off frequency

$$\Omega = \frac{2cy\sqrt{1 + y^2}}{n_0 d} \tag{10}$$

The film discussed is an example of so-called "gradient films", and modern nanotechnology can provide considerable differences between n_0 and n_m (e.g., [16], n_0 = 2.3, n_m = 1.47). As shown in [15], the existence of a cut-off frequency is linked to the concave nature of the index profile – for which p^2 is always positive – convex profiles leading eventually to negative values of p^2 and therefore to the absence of a cut-off frequency.

If the frequency of the incident wave ω is less than the heterogeneity-induced cut–off frequency of the film Ω (10), the wave was shown to tunnel through the film [17]. Using the boundary conditions, one can find the complex reflection and transmission coefficients of gradient dielectric film. These coefficients for travelling and tunneling regimes are given in Appendix. As compared with the case of traditional rectangular metallic barriers, tunneling through gradient photonic barriers possesses some peculiar features, which can be derived by means of formulae from [17], given in the Appendix, which are listed below:

(1) Unlike in the traditional tunneling effect, we have here a real positive value of the dielectric

(2) susceptibility (ε>0) in all the transparent nanofilm (0<*z*<*d*). Contrary to the usual strong reflection and exponential attenuation of the transmitted evanescent wave in a rectangular homogeneous barrier, the interference of forward and backward wave in such a gradient barrier was shown in [17] to provide an almost reflectionless and weakly attenuated energy flow in the tunneling regime for a large spectral range (Fig. 2a).

(3) The phase shift ϕ_t for the tunneling wave is strongly dispersive (Fig.2b).

(3) The phase ϕ_t at $\omega = \Omega$ possesses a discontinuity of π : the values ϕ_t in the tunneling range ($\omega \le \Omega$, $u \ge 1$) are positive, the values ϕ_t in the traveling range ($\omega \ge \Omega$, $u \le 1$) are negative. The phase shift ϕ_t can exceed in the tunneling spectral range the phase shift ϕ_0 accumulated by the wave with the same frequency ω traversing the same distance d in free space ($\phi_0 = \omega d / c$); this superluminal effect ($\phi_t > \phi_0$), as well as the subluminal effect ($\phi_t < \phi_0$) in the traveling range, are shown on Fig. 2b.

Keeping in mind these features, let us consider the tunneling of a femtosecond pulse with width t_0, carrier frequency ω_0 and amplitude E_0 (Fig.3)

$$E(t) = E_0 \sin\left(\frac{\pi t}{t_0}\right)\cos(\omega_0 t) \tag{11}$$

whose spectrum contains parts belonging to both traveling and tunneling spectral ranges. The pulse waveform after passage through the gradient barrier can be found by means of calculation of the Fourier transform $F(\omega)$ of the incident pulse (13) followed by the inverse Fourier transform of the product $T(\omega)F(\omega)$, where $T(\omega)$ is the complex transmission coefficient, shown on Fig.2. Depending on the tuning of carrier frequency ω_0 with respect to barrier cut-off frequency Ω the contributions of harmonics with sub- and superluminal phase shifts to the waveform of the transmitted pulse prove to be different.

3. Results and discussion

To illustrate this pulse dynamics, let us introduce the coordinate system moving together with the maximum of the pulse in vacuum with free space light velocity *c*; the points located behind (ahead) of this edge correspond to positive (negative) values of time. Fig. 3a – 3d, drawn in this coordinate system, illustrate the distortions of the fs pulses tunneling through a gradient barrier with Ω = 2.452 10^{15} rad/s, depending on their main carrier frequency. Transmitted pulses are shown on Fig.3a – 3d in the normalized units $E_1(t)/E_0$. Comparison of these graphs shows that the distortions of the tunneling pulses strongly depend upon the frequency detuning $\Delta = (\omega_0 - \Omega)/\Omega$ in the vicinity of cut-off frequency Ω.

Fig. 3a, related to a case of a still relatively large detuning Δ = -8$10^{-2}$, shows the beginning of a modulation of the envelope at the pulse wings and a small but noticeable pulse broadening. Designating, for shortness, the tunneling and freely propagating pulses as P_t and P_f respectively, one can see that the modulation at the front wing of P_t, decreasing in the direction of pulse propagation, has larger amplitudes than the ones of P_f at the same time. The same behavior is observed in the tailing edge of the pulse. The forerunning oscillations, formed essentially by strongly dispersed tunneling harmonics in the area t < 0, can be considered as superluminal precursors. In fig 3,b, the nearing of frequencies ω_0 and Ω ($\Delta = -4\,10^{-2}$), frequency ω_0 remaining in the tunneling spectral range, results in the growth of a clear superluminal precursor, with a peak amplitude of 0.7 and in the beginning of a pulse peak splitting down to the level 0.4. For ω_0 = Ω ($\Delta = 0$) the pulse is completely split (fig.3c), and two peaks with large amplitudes of about 0.58 are formed. Finally, shifting the carrier frequency ω_0 to the transparent spectral range ($\Delta = 4$ 10^{-2} > 0) weakens the reshaping effects: the remaining effects look at first sight very much alike those observed on fig 3,a. It is remarkable that these strong distortions are induced by a thin subwavelength gradient barrier (thickness d = 100 nm) with respect to the carrier infrared wavelengths ($\lambda \approx$740 - 800 nm).

Beside the amplitude effects discussed above, a careful inspection of the compared positions of amplitude extrema shows that in all figures, P_t shows over the time span represented a phase slip of π compared to P_f. The four figures however differ by the region over which this phase slip occurs. The case of fig 3,c is in this respect quite particular, since the phase slip occurs in a very small region around t = 0, the oscillations in both pulses being essentially in phase in the front part of the pulse (t<0) and out of phase in the tailing part (t >0). There is thus in this case a complete separation between the tunneling harmonics contributing only to the forerunning peak and the propagating one contributing only to the tailing peak. This emphasizes the essential role played by phase effects in the pulse reshaping. In this respect, it is worth recalling that the group velocity effects in such tunneling barriers have been investigated for continuous waves in [15-18]. It was shown that the group velocities in such gradient layers were always subluminal, and depended very little of the fact that the wave frequency was below or above the cut-off frequency. So the strong effects illustrated above are clearly related to the question of "phase time", a problem abundantly discussed in the literature concerning superluminal propagation [11-19]

At first sight, modulation of pulse edges resembles the classical effect of formation of Sommerfeld - Brillouin precursors, which are known to arise in the course of pulse propagation in dispersive transparent media without tunneling spectral range [20]. However, the speed of these precursors is subluminal, the path of their formation Z must be long enough (Z>> c t_0) and they are concentrated at the pulse edges [21]. On the contrary, the salient features of phenomena we discuss here are:

(1) The precursors, formed on the leading edge of pulse due to pulse tunneling through gradient photonic barrier, accompanied by its lengthening, exhibit a superluminal displacement.

(2) Owing to phase discontinuity shown in (Fig.2b), providing the rapid displacement of tunneling harmonics, a drastic pulse reshaping is obtained over subwavelength distances.

(3) Depending on the frequency detuning Δ , a complete splitting of the pulse peak can be achieved.

It should be noted that the results predicted above are well in the field of today's experimental possibilities. Concerning the nanofilms, it has been mentioned that such index variations have already been realized [17], and controlling during the nanofilm's growth its index profile through polarimetry is now a classical technique. As for the deformation, and particularly the splitting, to which the femtosecond pulse would be subjected, they are large enough to be measured using the traditional femtosecond pulse characterization techniques. Of course, since the question of phase shifts in the transmitted pulse is central to the problem, it would be extremely interesting to use the most sophisticated of these techniques, as SPIDER [22], allowing to measure simultaneously over the whole pulse both its amplitude and phase spectrum.

Such an experiment, which appears clearly feasible on account of presently obtainable gradient nanolayers, will modify essentially the standard concepts of tunneling:

(1) Using of these non–attenuative modes for effective energy transmission in the tunneling regime removes the problem of measurement of exponentially damping evanescent waves;

(2) Optically thick non–transparent layers, usually considered as a tool for obtaining superluminal tunneling times (as in the Hartman geometry [19]), are not in need for the gradient photonic barriers of subwavelength thickness under discussion;

(3) Since the spectra for evanescent waves were obtained due to exact solutions of Maxwell equations, similar superluminal phase effects can arise in other spectral ranges of EM waves, e.g., for microwaves [23].

Before concluding, it is worth trying to relate the observed phenomena to some more traditional ones in optics (linear of not) of femtosecond pulses: two of them can give rise to strong reshaping of such pulses. The first one is group velocity dispersion, which also occurs here since such gradient nanolayers are dispersive systems. But, as we already said, group velocity is not so sensitive to the exact position of the pulse carrier frequency compared to the cut-off frequency contrary to what we calculate. The second such effect is phase-modulation, which seems in view of fig 2 a much more probable explanation, given the very strong variations of the phase shift in the domain where the strong effects are observed (the coefficient $\partial\varphi/\partial\omega$ taking an infinite value in u=1). Therefore we suggest that the effects discussed here can be assigned to the strong spectral phase modulation provided by such gradient nanolayers.

4 - Conclusion

In conclusion, let us stress out that the phase phenomena discussed above are specific to evanescent EM wave pulses in transparent gradient media, where the spatial and temporal variations of EM fields are not connected by the classical dispersive equations. Owing to the strong heterogeneity-induced dispersion of the gradient barrier and to the phase discontinuity of tunneling wave, the distorted transmitted pulse, some part of which exhibiting a superluminal dynamics, differs drastically from the initial incident pulse. It is essential that the Einsteinian formulation "The velocity of light in vacuum cannot depend upon the velocity of its source" [24] is not violated by this experiment. First, we showed above that, on the whole, the energy propagation stays subluminal but, more important perhaps, the above results are obtained in a situation where we showed earlier that the group velocity of all the harmonics forming the pulse is also subluminal inside the film. This emphasizes again the essential role of the phase discontinuities at the interfaces in the effects discussed above. Finally, the strong pulse reshaping accompanied by the possibility of superluminal outstripping of some part of a pulse as compared with the free motion of pulse in vacuum may become interesting for applications.

5. Appendix : useful formulae.

Formulae for calculation of complex transmission coefficient $T = |T|\exp(i\phi_t)$ through the barrier are given for the tunneling regime $(u \geq 1)$ by formulae (1A) – (4A); for the traveling regime $(u \leq 1)$ - by formulae (5A) – (8A):

Tunneling regime:

$$|T|^2 = \frac{4nn_e^2\left(1-t^2\right)}{|\aleph|^2}; \tag{1A}$$

$$|\aleph|^2 = \left[t\left(n - \frac{\gamma^2}{4} - n_e^2\right) + \gamma n_e\right]^2 + (n+1)^2\left(n_e - \frac{\gamma t}{2}\right)^2; \tag{2A}$$

$$\cos\phi_t = \frac{(n+1)\left(n_e - \frac{\gamma t}{2}\right)}{|\aleph|}; \qquad \sin\phi_t = \frac{t\left(n - \frac{\gamma^2}{4} - n_e^2\right) + \gamma n_e}{|\aleph|}; \qquad n_e^2 = n_0^2\left(u^2 - 1\right) \tag{3A}$$

$$\gamma = \frac{2n_0 uy}{\sqrt{1+y^2}}; \qquad t = th\left(l\sqrt{1-\frac{1}{u^2}}\right); \quad l = \ln\left(\frac{y_+}{y_-}\right); \quad y_\pm = \sqrt{1+y^2} \pm y; \quad y = \sqrt{\frac{n_0}{n_{\min}} - 1}; \tag{4A}$$

Traveling regime:

$$|T|^2 = \frac{4nn_e^2\left(1+t^2\right)}{|\Gamma|^2}; \tag{5A}$$

$$|\Gamma|^2 = \left[t\left(n - \frac{\gamma^2}{4} + n_e^2\right) + \gamma n_e\right]^2 + (n+1)^2\left(n_e - \frac{\gamma t}{2}\right)^2; \quad t = tg\left(l\sqrt{\frac{1}{u^2} - 1}\right) \tag{6A}$$

$$\cos\phi_t = \frac{(n+1)\left(n_e - \frac{\gamma t}{2}\right)}{|\Gamma|}; \quad \sin\phi_t = \frac{t\left(n - \frac{\gamma^2}{4} + n_e^2\right) + \gamma n_e}{|\Gamma|}; \quad n_e^2 = n_0^2\left(1 - u^2\right) \tag{7A}$$

where y, γ and l are defined in (4A).

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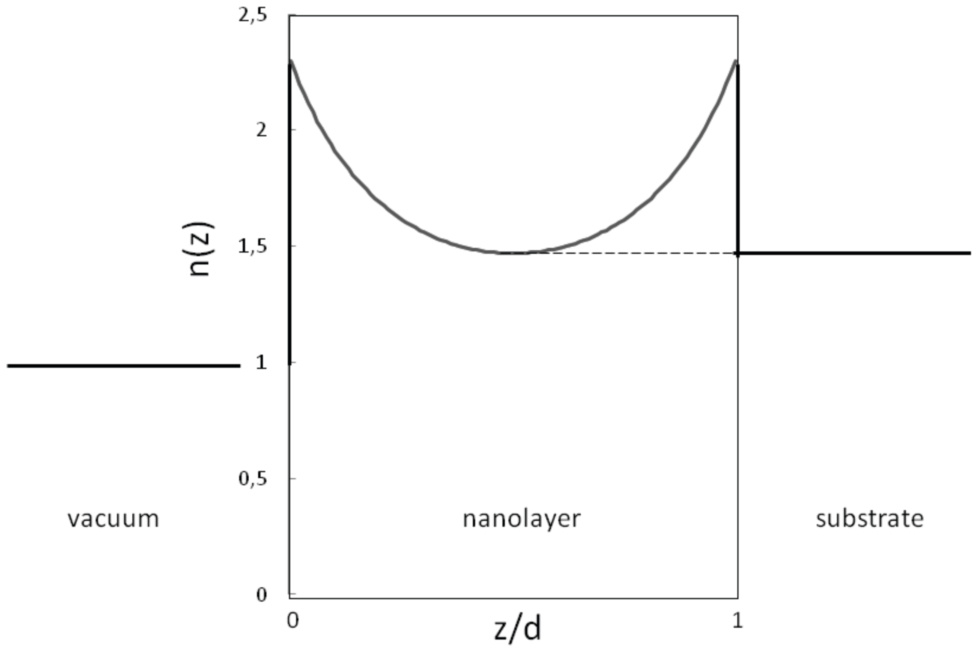

Fig.1. Normalized profile of dielectric susceptibility U^2 (1) inside the gradient photonic barrier, providing the tunneling of EM wave through barrier with thickness d, located on the substrate; z/d is the normalized coordinate across the barrier.

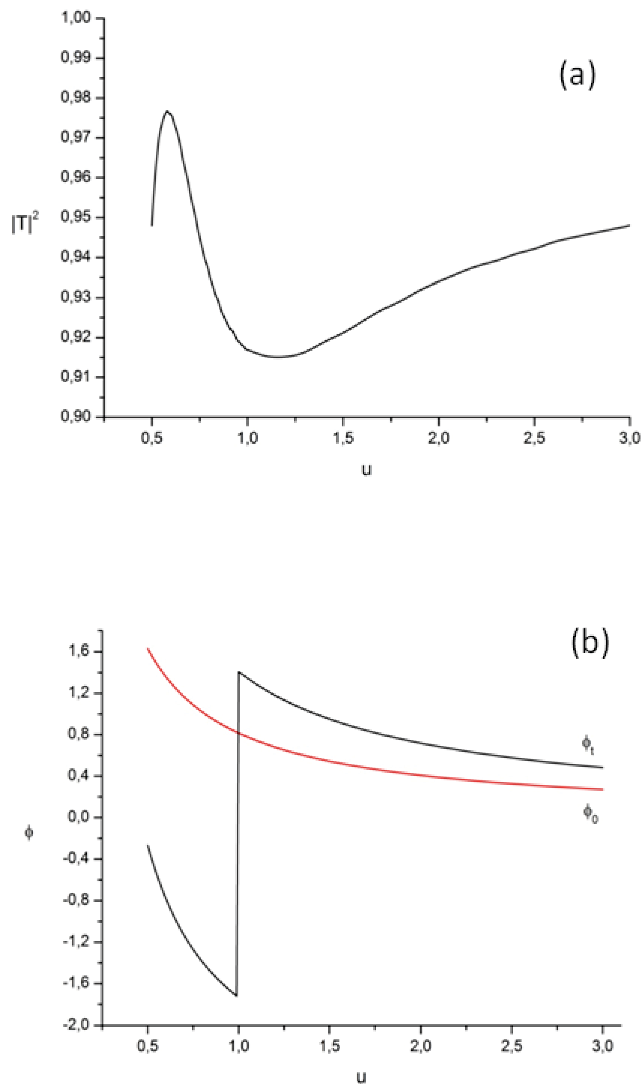

Fig.2. Amplitude-phase spectra of tunneling through the photonic barrier, shown on Fig.1; the values of barrier parameters are: $n_0 = 2.3$, $n_{min} = n = 1.47$, $d = 100$ nm. (Fig.2a) – ransmission coefficient with respect to energy $|T|^2$, Fig.2b – phase shift of transmitted wave ϕ_t , ϕ_0 - phase shift of wave, freely propagating in a free space on the same distance d, u = Ω / ω - normalized frequency.

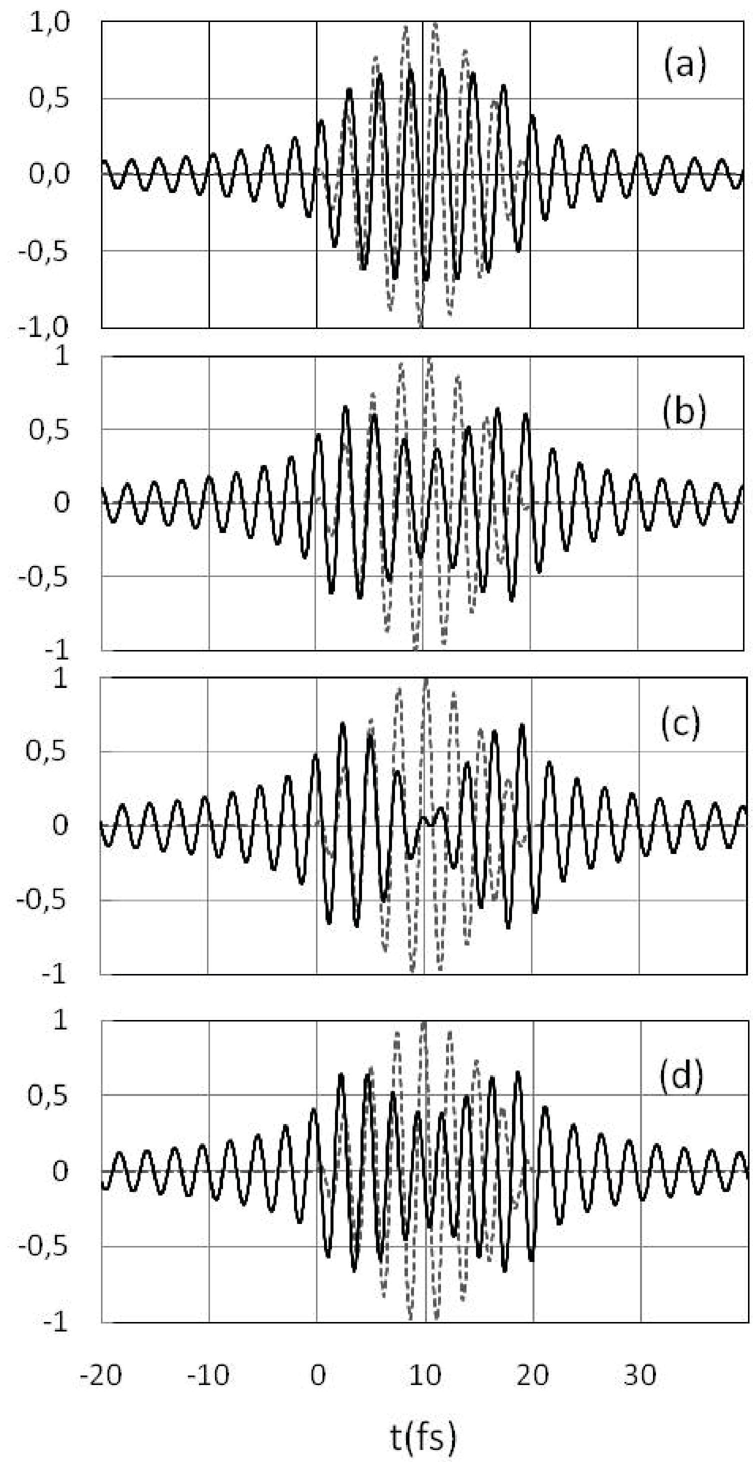

Fig.3. Normalized temporal envelops of fs pulse (11) (t_0=20фс), propagating in a free space (dotted lines) and tunneling through the gradient barrier (solid lines), Ω = 2.45 10^{15} rad/s; carrier frequencies ω_0 and detunings Δ at figures 3a – 3d are ω_0 = 2.25, 2.35, 2.45 and 2.55 10^{15} rad/s and $\Delta = -8.16\ 10^{-2}$, -4.08 10^{-2}, 0 and 4.08 10^{-2} respectively. Superluminal precursors are forming in the area t < 0.