Design of new phase shifting algorithms according to interferometer sensitivities

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ABSTRACT

Correct metrological calibration is the key to avoiding the main systematic error sources in an interferometer configuration in order to obtain high resolutions and repeatability. The calibration procedure can consist of the acquisition of two fringe patterns with a known phase shift between them, which means the correct adjustment of this phase difference fits the system. Differential phase shifting algorithms, built by combining two known phase shifting algorithms in a non-linear way, obtain this phase difference directly. In this sense, the two-dimensional characteristic polynomial is an innovative tool for qualitatively characterizing the properties of the differential phase shifting algorithms and, particularly, for determining the accuracy of the system. In previous works, it was demonstrated that sensitivities are inherited from precursor phase shifting algorithms. This mechanism of error propagation allows us to design new differential phase shifting algorithms according to standard requirements in each system in order to cancel or at least minimize this known source of errors.

Keywords: differential phase shifting algorithms, characteristic polynomial, calibration, interferometry.

1. INTRODUCTION

It is known [1-4] that interferometer configurations are affected by systematic and random errors that affect the measurement process. Current metrology is not able to obtain the value of a magnitude with total accuracy and precision, although it is possible to significantly minimize the influence of the main sources of systematic errors in the final measurand. In this sense, the most widely used tool for acquiring the phase of a fringe pattern is the phase shifting algorithm (PSA) because of its well-known properties. PSAs retrieve the optical phase by means of a combination of M α-shifted fringe images $s_m(\mathbf{r},\phi,\alpha_m)$,

$$s_m(\mathbf{r},\phi,\alpha_m) = \sum_{k=0}^{\infty} a_k(\mathbf{r})\cos[k(\phi+\alpha_m)] = \sum_{k=-\infty}^{\infty} \frac{a_k(\mathbf{r})}{2} e^{jk(\phi+\alpha_m)} \tag{1}$$

where a_k is the harmonic contribution, weighted with the coefficients n_m and d_m of the corresponding PSA in the argument of the arctan function [5,6]

$$\phi(\mathbf{r}) = \arctan \frac{\sum_{m=1}^{M} n_m s_m(\mathbf{r},\phi,\alpha_m)}{\sum_{m=1}^{M} d_m s_m(\mathbf{r},\phi,\alpha_m)} \tag{2}$$

Since this pattern can be greatly affected by errors in the phase modulator, it is necessary to carry out a previous calibration of this modulator. It can be done by the acquisition of consecutive M images with a known phase shift and to retrieve the calculated α by means of a combination of the irradiance values $s_m(\mathbf{r},\phi,\alpha_m)$ with the coefficients (n'_m, d'_m) that depend on the PSA [7-10]:

$$\alpha = f\left(n'_m, d'_m, s_m(\mathbf{r},\phi,\alpha_m)\right) \tag{3}$$

On the other hand, with two fringe patterns with a nominal control shift δ between the first M images of the original pattern $s_m(\mathbf{r},\phi,\alpha_m)$ and the δ-shifted P images of the modified one $t_p(\mathbf{r},\phi+\delta,\beta_p)$, with a β displacement between the p and (p+1) images with g harmonics weighted with the coefficients b_g,

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$$t_p\left(\mathbf{r},\phi+\delta,\beta_p\right)=\sum_{g=0}^{\infty} b_g(\mathbf{r})\cos\left[g\left(\phi+\delta+\beta_p\right)\right]=\sum_{g=-\infty}^{\infty}\frac{b_g(\mathbf{r})}{2}e^{jg\left(\phi+\delta+\beta_p\right)} \tag{4}$$

it is also possible to undertake a calibration procedure using the differential phase shifting algorithms (DPSA) [11] that combine two PSAs, which are not necessarily equal, in a non-linear way, and that can also be expressed as a quantification process of the auxiliary mathematical function, product of the two patterns, with the sampling coefficients (n_m, d_m) and (n_p, d_p)

$$\begin{aligned}\delta &= \arctan\frac{\sum_{m=1}^{M} d_m s_m\left(\mathbf{r},\phi,\alpha_m\right)\sum_{p=1}^{P} n_p t_p\left(\mathbf{r},\phi+\delta,\beta_p\right)-\sum_{m=1}^{M} n_m s_m\left(\mathbf{r},\phi,\alpha_m\right)\sum_{p=1}^{P} d_p t_p\left(\mathbf{r},\phi+\delta,\beta_p\right)}{\sum_{m=1}^{M} n_m s_m\left(\mathbf{r},\phi,\alpha_m\right)\sum_{m=1}^{M} d_m s_m\left(\mathbf{r},\phi,\alpha_m\right)+\sum_{p=1}^{P} n_p t_p\left(\mathbf{r},\phi+\delta,\beta_p\right)\sum_{p=1}^{P} d_p t_p\left(\mathbf{r},\phi+\delta,\beta_p\right)}= \\ &= \arctan\frac{\sum_{m=1}^{M}\sum_{p=1}^{P} n_{m,p} s_m\left(\mathbf{r},\phi,\alpha_m\right)t_p\left(\mathbf{r},\phi+\delta,\beta_p\right)}{\sum_{m=1}^{M}\sum_{p=1}^{P} d_{m,p} s_m\left(\mathbf{r},\phi,\alpha_m\right)t_p\left(\mathbf{r},\phi+\delta,\beta_p\right)}\end{aligned} \tag{5}$$

This work shows the sensitivities associated to each of two different ways to calibrate an interferometer using as an example the Schwider-Hariharan PSA [12,13] and the Schwider-Hariharan DPSA respectively.

2. CALIBRATION OF THE ADDITIONAL PHASE WITH ONE PSA

The two main systematic errors [14] in an interferometer are the presence of undesired harmonics in the irradiance values (the ideal patterns should have a_k=0 when k>1 and b_g=0 when g>1)

$$Es_m(\mathbf{r},\phi)=\sum_{k=2}^{\infty} a_k(\mathbf{r})\cos\left[k\left(\phi(\mathbf{r})+\alpha_m\right)\right] \tag{6a}$$

$$Et_p(\mathbf{r},\phi+\delta)=\sum_{g=2}^{\infty} b_g(\mathbf{r})\cos\left[g\left(\phi(\mathbf{r})+\delta+\beta_p\right)\right] \tag{6b}$$

and the phase shift errors [15] introduced by the modulator that can be expressed for each pattern as:

$${}^{E}\alpha_m=\alpha_m+E\alpha_m=\alpha_m+\sum_{q=1}^{\infty}\varepsilon_q\frac{\alpha_m^q}{q\pi^{q-1}} \tag{7a}$$

$${}^{E}\beta_p=\beta_p+E\beta_p=\beta_p+\sum_{r=1}^{\infty}\chi_r\frac{\beta_p^r}{r\pi^{r-1}} \tag{7b}$$

With errors $E\alpha_m$, for the original pattern, and $E\beta_p$, for the modified one, quantified by the q^{th} and r^{th} terms ε_q and χ_r.
This section shows the way the original pattern is affected by these two main systematic errors and a similar analysis is valid for the modified one.
An initial approximation to the behavior of the PSA can be made in the Fourier space associating a characteristic polynomial (CP) to the calculation of the additional phase in a similar way that it is associated to retrieve the phase [16-21]. In this case, the drawback is that the additional phase is not always retrieved via the same function and the CP has to be thought ad hoc for each PSA. For instance, if the algorithm to recover the additional phase can be expressed via arctan function the CP should be:

$$V(\alpha)=\sum_{m=1}^{M}\left(d'_m+jn'_m\right)s_m\left(\mathbf{r},\phi,\alpha_m\right)=\sum_{k=-\infty}^{\infty}\left[a_k e^{jk\phi}\sum_{m=1}^{M}\left(d'_m+jn'_m\right)e^{jk\alpha_m}\right]=\sum_{k=-\infty}^{\infty}a_k e^{jk\phi}P\left(e^{jk\alpha}\right) \tag{8}$$

thus the multiplicity and localization of the roots of this $P[\exp(jk\alpha)]$ informs us of its sensitivity to these main error sources. For calibration purposes it can be considered $\phi=0$ to simplify the experiment; there is no difference in the optical path between the two arms of the interferometer. If there are considered miscalibration errors, in addition to the presence of undesired harmonics in the signal, equation (8) can be rewritten as

$$
\begin{aligned}
V(\alpha) &= \sum_{k=-\infty}^{\infty} a_k \sum_{m=1}^{M} \left(d'_m + jn'_m\right) e^{jk\left(\alpha_m + \sum_{q=1}^{\infty} \varepsilon_q \frac{\alpha_m^q}{q\pi^{q-1}}\right)} = \\
&= \sum_{k=-\infty}^{\infty} a_k \sum_{m=1}^{M} \left(d'_m + jn'_m\right) e^{jk\alpha_m} \left[1 + jk \sum_{q=1}^{\infty} \varepsilon_q \frac{\alpha_m^q}{q\pi^{q-1}}\right] = \\
&= \sum_{k=-\infty}^{\infty} a_k \left[P\left(e^{jk\alpha}\right) + \sum_{q=1}^{\infty} \left[jke^{jk\alpha}\right]^q \frac{\varepsilon_q \alpha^q}{q\pi^{q-1}} \frac{d^q P\left(e^{jk\alpha}\right)}{d\left(e^{jk\alpha}\right)^q} \right]
\end{aligned} \tag{9}
$$

The insensitivity to undesired harmonics (k>1) demands that $P[\exp(jk\alpha)]$ is cancelled at these frequencies. Regarding phase detuning errors, the double roots of $\exp(-j\alpha)$ must appear, and not for $\exp(j\alpha)$ since the fundamental harmonic must be detected.

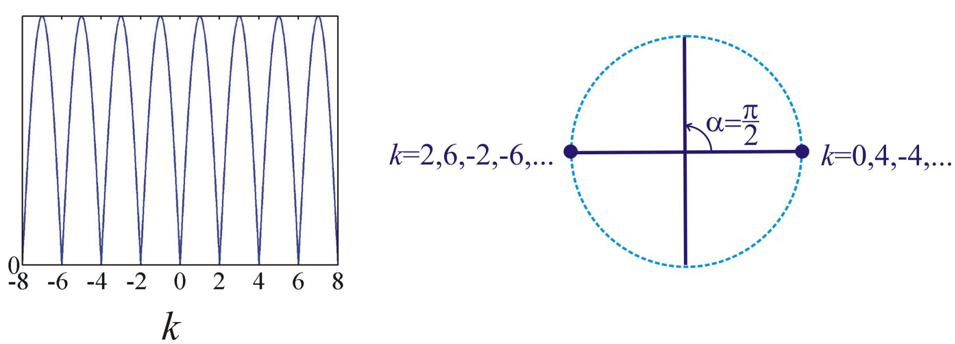
On the other hand, a quantitative analysis provides the magnitude of these errors by means a linearization process [22]. The generic expression for a PSA of M shifts is

$$E\alpha = \sum_{m=1}^{M} \frac{\partial \alpha}{\partial s_m} \frac{\partial s_m}{\partial \alpha_m} E\alpha_m + \sum_{m=1}^{M} \left(\frac{\partial \alpha}{\partial s_m}\right) Es_m \tag{10}$$

As an example of these two analyses, according to equation (3), is shown the Schwider-Hariharan PSA, which requires the value of the phase shift with a linear combination of its π/2 shifted irradiance values in the argument of the arccos function [12]:

$$\alpha = \arccos \frac{s_1 - s_5}{2s_2 - 2s_4} \tag{11}$$

In this case, it is easy to express equation (11) as a complex number. Figure 1 shows module and characteristic diagram for Schwider-Hariharan patterns where it can be seen that it detects all the even harmonics and, unlike the Schwider-Hariharan PSA, is detuning sensitive.



$$P\left(e^{jk\alpha}\right) = \left(1 - e^{j4k\alpha}\right) + j\sqrt{\left(2e^{jk\alpha} - 2e^{j3k\alpha}\right)^2 - \left(1 - e^{j4k\alpha}\right)^2}$$

Figure 1. Module and characteristic diagram for the calculation of the additional phase for Schwider-Hariharan patterns.

The analytical expression of the discrepancy in the nominal value to first (q=1) and second order (q=2) and the non-sinusoidal profile of the signal with the presence of second (k=2) and third order harmonics (k=3) for the Schwider-Hariharan PSA is [22]

$$E\alpha = \frac{\pi\varepsilon_1}{2\sin^2\alpha} + \frac{\pi\varepsilon_2}{8}\frac{\cos\alpha\cos\phi}{\sin^2\alpha\sin\phi} - \frac{a_3\cos\alpha\sin 3\phi}{a_1\sin\phi\sin^2\alpha} \tag{12}$$

which validates the qualitative results and shows a dependency with sinusoidal functions in the denominator that could introduce discontinuities in the recovered phase shift.

3. CALIBRATION OF THE ADDITIONAL PHASE WITH TWO PSAS

DPSAs inherit their properties from their precursor PSAs in an arithmetic way so the existing knowledge on the behavior of each precursor PSA can be used to design especially insensitive DPSAs [23-25]. In this context, the phase δ is the calibration phase so, the more insensitive the PSAs involved in the process are, the more accurate the calibration of the interferometer is.

In previous works [24,25], it was shown that a qualitative analysis can be made associating a complex number to the calculation of the phase difference with a DPSA in a way that the roots of the two dimensional characteristic polynomial (TDCP) $P(e^{jk\alpha},e^{jg\beta})$ of the DPSA informs us of the presence of harmonics or phase shifting errors:

$$\begin{aligned} V(\delta) &= \sum_{m=1}^{M}\sum_{p=1}^{P}\left(d_{m,p} + jn_{m,p}\right)s_m\left(r,\phi,\alpha_m\right)t_p\left(r,\phi+\delta,\beta_p\right) = \\ &= \sum_{k=-\infty}^{\infty}\sum_{g=-\infty}^{\infty} a_k b_g e^{j[g\delta+(g+k)\phi]}\sum_{m=1}^{M}\sum_{p=1}^{P}\left(d_{m,p} + jn_{m,p}\right)e^{jk\alpha_m}e^{jg\beta_p} = \\ &= \sum_{k=-\infty}^{\infty}\sum_{g=-\infty}^{\infty} e^{j[g\delta+(g+k)\phi]}P\left(e^{jk\alpha},e^{jg\beta}\right) \end{aligned} \tag{13}$$

The calculation of this TDCP can be simplified if it is expressed as a multiplication of the CPs of its precursors, taking into account that to recover the phase difference the original pattern has to obtain $-\phi$

$$P\left(e^{jk\alpha},e^{jg\beta}\right) = \sum_{m=1}^{M}\sum_{p=1}^{P}\left(d_m - jn_m\right)\left(d_p + jn_p\right)e^{jk\alpha(m-1)}e^{jg\beta(p-1)} = P^*\left(e^{-jk\alpha}\right)P\left(e^{jg\beta}\right) \tag{14}$$

If it is considered that the two main systematic errors affect the signals, equation (14) is modified as follows

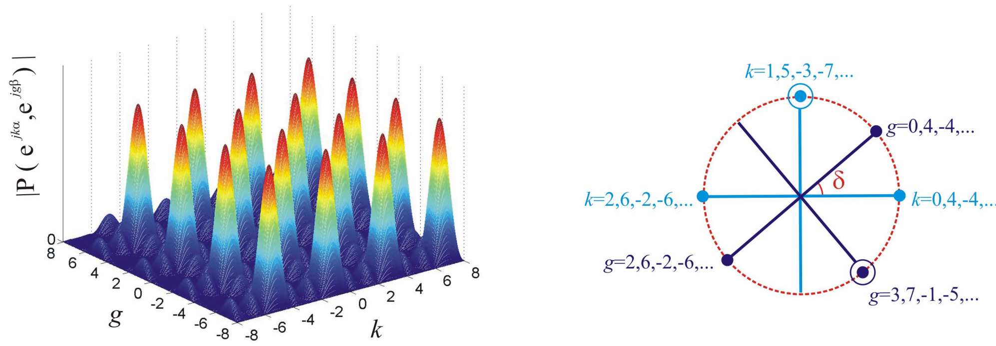
$$\begin{aligned} V(\delta) &= \sum_{k=-\infty}^{\infty}\sum_{g=-\infty}^{\infty}\frac{a_k b_g}{4}\, e^{j[g\delta+(g-k)\phi]}\sum_{m=1}^{M}\sum_{p=1}^{P}\left(d_{m,p} + jn_{m,p}\right)e^{jk\,{}^{E}\alpha_m}e^{jg\,{}^{E}\beta_p} = \\ &= \sum_{k=-\infty}^{\infty}\sum_{g=-\infty}^{\infty}\frac{a_k b_g}{4}\, e^{j[g\delta+(g-k)\phi]}\left\{P\left(e^{jk\alpha},e^{jg\beta}\right) + \right. \\ &+ j\left[k\sum_{q=1}^{\infty}\frac{\varepsilon_q\alpha^q}{q\pi^{q-1}}\frac{\partial^q P\left(e^{jk\alpha},e^{jg\beta}\right)}{\partial\left(e^{jk\alpha}\right)^q}e^{jk\alpha} + g\sum_{r=1}^{\infty}\frac{\chi_r\beta^r}{r\pi^{r-1}}\frac{\partial^r P\left(e^{jk\alpha},e^{jg\beta}\right)}{\partial\left(e^{jg\beta}\right)^r}e^{jg\beta}\right] - \\ &\left. - kg\sum_{q=1}^{\infty}\sum_{r=1}^{\infty}\frac{\varepsilon_q\alpha^q}{q\pi^{q-1}}\frac{\chi_r\beta^r}{r\pi^{r-1}}\frac{\partial^r}{\partial}\frac{\partial^r P\left(e^{jk\alpha},e^{jg\beta}\right)}{\partial\left(e^{jg\beta}\right)^r}e^{jk\alpha}e^{jg\beta}\right\} \end{aligned} \tag{15}$$

So, the only harmonic that should be detected is the reference one, $(k,g)=(-1,1)$, the rest must be null in order to avoid the presence of undesired harmonics. Accepting that the patterns should not have detuning errors demands mathematically that $\exp(j\alpha)$ and $\exp(-j\beta)$ be double roots. Likewise, in order to avoid error in the q and r order phase shifts, their derivatives of this order must be null.

The Schwider-Hariharan DPSA is used as an example to illustrate the calibration with two patterns:

$$\delta = \arctan\frac{\left(2s_3 - s_1 - s_5\right)\left(2t_2 - 2t_4\right) - \left(2s_2 - 2s_4\right)\left(2t_3 - t_1 - t_5\right)}{\left(2s_3 - s_1 - s_5\right)\left(2t_3 - t_1 - t_5\right) + \left(2s_2 - 2s_4\right)\left(2t_2 - 2t_4\right)} \tag{16}$$

Figure 2 shows the Schwider-Hariharan TDCP with the representation of its amplitude and characteristic diagram. It can be seen, for example, that this DPSA is detuning error compensated (as its PSA precursor), since at k=1 and g=-1 it has double roots, or that the even harmonics of δ with k=g are not detected



$$P\left(e^{jk\alpha}, e^{jg\beta}\right) = \left(e^{jk\alpha}+1\right)\left(e^{jk\alpha}-1\right)\left(e^{jk\alpha}-j\right)^2\left(e^{jg\beta}+1\right)\left(e^{jg\beta}-1\right)\left(e^{jg\beta}+j\right)^2$$

Figure 2. Module and characteristic diagram for Schwider-Hariharan TDCP.

In the same way, its quantitative error can be calculated as an addition of the phase errors of its precursors or, in other words, as a consequence of the linearization process:

$$E\delta(\mathbf{r}) = \sum_{m=1}^{M}\left(\frac{\partial\delta}{\partial s_m}\right)\left(\frac{\partial s_m}{\partial\alpha_m}\right)E\alpha_m + \sum_{p=1}^{P}\left(\frac{\partial\delta}{\partial t_p}\right)\left(\frac{\partial t_p}{\partial\beta_p}\right)E\beta_p + \sum_{m=1}^{M}\left(\frac{\partial\delta}{\partial s_m}\right)Es_m + \sum_{p=1}^{P}\left(\frac{\partial\delta}{\partial t_p}\right)Et_p \quad (17)$$

For the Schwider-Hariharan DPSA, the magnitude of the phase shift error is calculated to first (q=1 and r=1) and second order (q=2 and r=2) and the presence of harmonics to second (k=2 and g=2) and third order (k=3 and g=3)

$$E\delta = \frac{\pi\varepsilon_2}{8}\sin\delta\sin(2\phi+\delta) + \frac{a_3}{a_1}\sin 4\phi - \frac{b_3}{b_1}\sin(4\phi+4\delta) \quad (18)$$

In this case, it can be observed that there is a periodical nature in the error with a sinusoidal dependence in multiples of the phases of both patterns.

4. DESIGN OF NEW ALGORITHMS: SCHWIDER-HARIHARAN – SYMMETRICAL 6+1 DPSA

DPSAs show interesting compensatory capabilities inherited from their precursor PSAS that can help in the design of new tailored DPSAs. In this sense, the TDCP is an efficient tool for phase error examination that allows us to obtain information about error calibration and undesired harmonic contribution in the signal.

It is important to note that the assignation of a particular PSA to the original pattern or to the modified one can alter the sensitivities of the DPSA. Examples can be seen in the Schwider-Hariharan – Symmetrical 6+1 DPSA

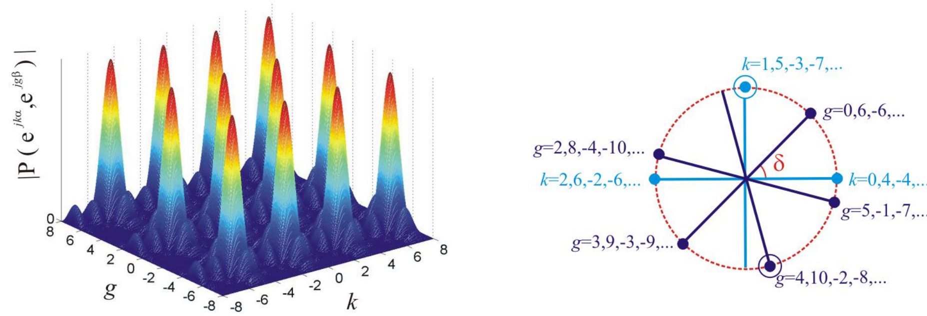
$$\delta = \arctan\frac{(2s_3-s_1-s_5)(t_2+t_3-t_5-t_6)+\sqrt{3}(2s_2-2s_4)(t_1+t_2-t_3-2t_4-t_5+t_6+t_7)}{-\sqrt{3}(2s_2-2s_4)(t_2+t_3-t_5-t_6)+(2s_3-s_1-s_5)(t_1+t_2-t_3-2t_4-t_5+t_6+t_7)} \quad (19)$$

and the Symmetrical 6+1 – Schwider-Hariharan DPSA.

$$\delta = \arctan\frac{(2t_2-2t_4)(s_1+s_2-s_3-2s_4-s_5+s_6+s_7)+\sqrt{3}(s_2+s_3-s_5-s_6)(2t_3-t_1-t_5)}{-\sqrt{3}(2t_2-2t_4)(s_2+s_3-s_5-s_6)+(2t_3-t_1-t_5)(s_1+s_2-s_3-2s_4-s_5+s_6+s_7)} \quad (20)$$

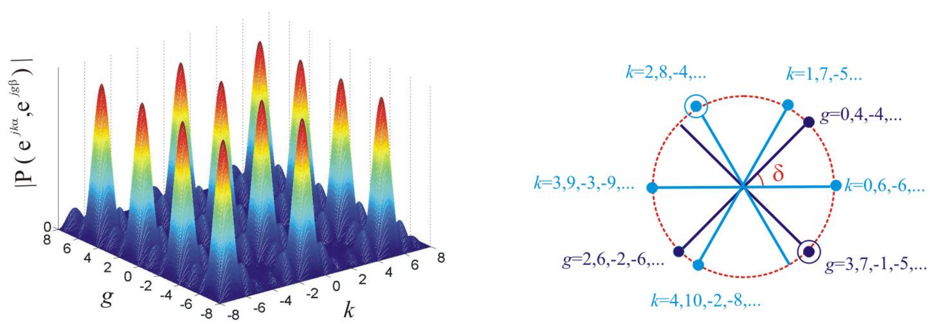
Both of these are designed with the combination of one PSA that is insensitive to detuning, Schwider-Hariharan PSA, and one PSA that is very insensitive to harmonic errors since the first harmonic Symmetrical 6+1 PSA detects is the fifth.

Figure 3 and Figure 4 show that both DPSAs detect the signal at (k,g)=(-1,1) so it can recover δ but at (k,g)=(1,-1) they do not have the double root that would indicate insensitivity to detuning. It would be necessary for both precursors to be insensitive to detuning to transfer this property but only the Schwider-Hariharan PSA is. On the other hand, the harmonics that each DPSA detects are different, as can be seen in the amplitude plot and characteristic diagram of Figure 3 and Figure 4. For instance, the Schwider-Hariharan - Symmetrical 6+1 DPSA detects (k,g)=(5,5) but not (k,g)=(-5,-5) and the Symmetrical 6+1 – Schwider-Hariharan DPSA detects (k,g)=(-5,-5) but not (k,g)=(5,5).



$$P\left(e^{jk\alpha}, e^{jk\beta}\right) = \left(e^{jk\alpha}-1\right)\left(e^{jk\alpha}+1\right)\left(e^{jk\alpha}-j\right)^2\left(e^{jg\beta}-1\right)\left(e^{jg\beta}+1\right)\left(e^{jg\beta}+e^{j\frac{\pi}{3}}\right)^2\left(e^{jg\beta}+e^{j\frac{2\pi}{3}}\right)\left(e^{jg\beta}-e^{j\frac{2\pi}{3}}\right)$$

Figure 3. Module and characteristic diagram for Schwider-Hariharan - Symmetrical 6+1 TDCP.



$$P\left(e^{jk\alpha}, e^{jk\beta}\right) = \left(e^{jk\alpha}-1\right)\left(e^{jk\alpha}+1\right)\left(e^{jk\alpha}-e^{j\frac{\pi}{3}}\right)\left(e^{jk\alpha}-e^{j\frac{2\pi}{5}}\right)\left(e^{jk\alpha}-e^{j\frac{4\pi}{3}}\right)\left(e^{jg\beta}-1\right)\left(e^{jg\beta}+1\right)\left(e^{jg\beta}+j\right)^2$$

Figure 4 Module and characteristic diagram for Symetrical 6+1 - Schwider-Hariharan TDCP.

5. EXPERIMENTAL DESIGN

$\pi/2$ shifted images were acquired with a phase-shifting interferometer. Figure 5 shows these images grouped in two patterns, the original and the modified one, to calculate the phase shift for each pattern, α and β, with equation (11) and the phase difference between the original pattern and the modified one, δ, with equation (16). Both calculations are a means to estimate the accuracy and precision of the phase shifter of the interferometer.

It can be seen that in both cases equation (11) recovers a sinusoidal pattern with marked discontinuities according with its calculated error, equation (12), and the behavior of δ shows a smoother behavior. Moreover, the standard deviation for δ (2º) is rather smaller than for α (9º) or β (11º). In consequence, for Schwider-Hariharan patterns the calibration with two patterns recovers a more accurate additional phase.

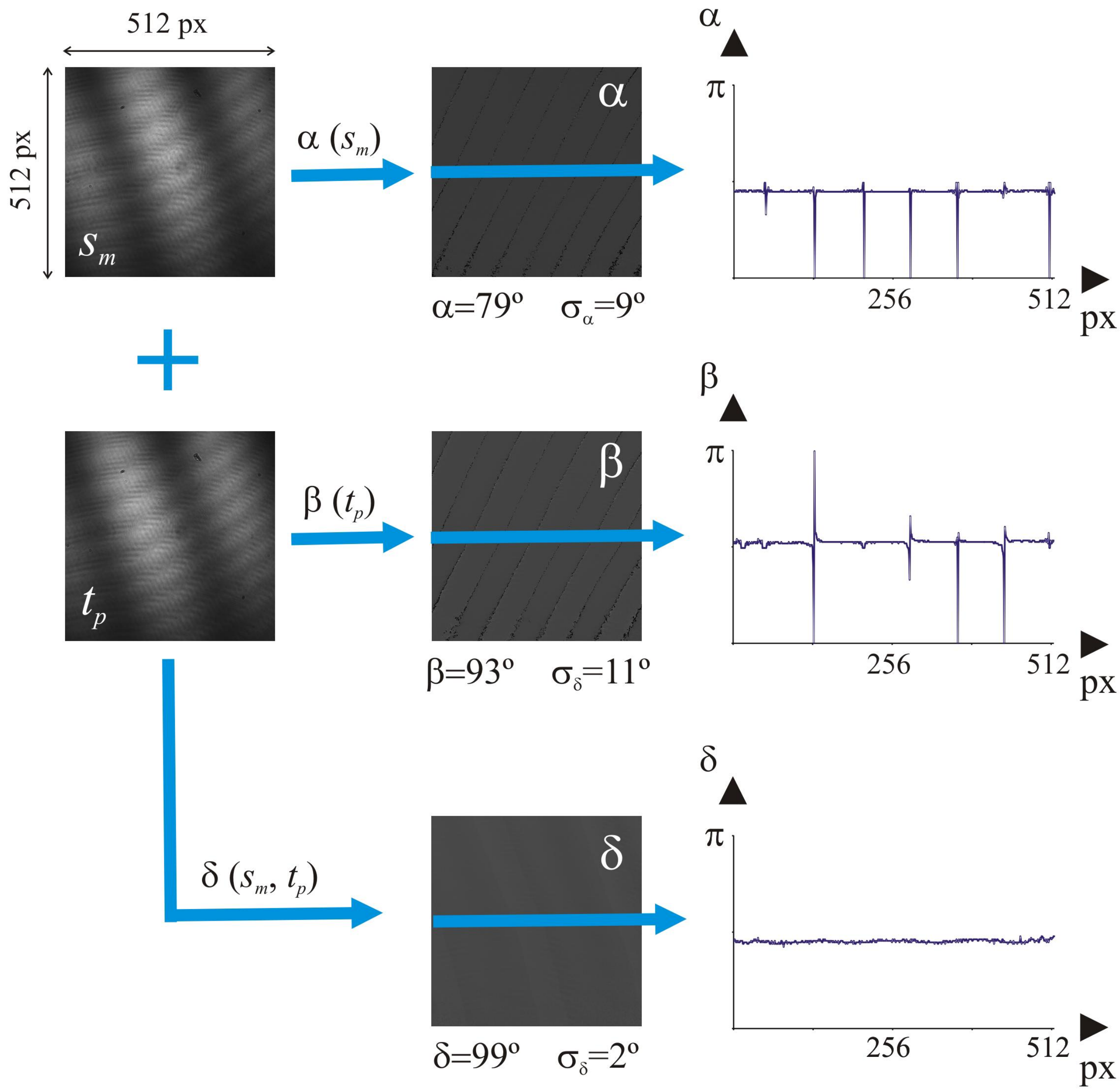


Figure 5 Original and modified Schwider-Hariharan patterns with their additional phases calculated with equation (11) and their difference phase between patterns calculated with equation (16).

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