

# Microwave Frequency Detector Using a 4x4 Butler Matrix

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## ABSTRACT

The paper presents an analysis of a microwave interferometer with a multi-ports architecture with one input and four outputs. The tested system consisted of two main parts: a subsystem consisting of a power divider and two transmission lines of different electric length - responsible for developing two signals whose phase difference will depend on the frequency of the input signal. The second part of the tested system consisted of a six-port element - a 4 x 4 Butler matrix, whose task was to develop four signals. The output amplitude depends on the phase difference of the signals fed to the inputs of this six-ports. Frequency detector tests were carried out by calculations in the MATLAB computer program in a very wide band  $f_g/f_d=4$ . The system response was analyzed for different combinations pair of an input ports of a six-port system and for different values of coupler coupling coefficients. During the analysis it was shown that the parameters of such an interferometer depend on which ports pair is excited as well as on the value of the coupling factor of the couplers. It is important to specify the system parameters in order to make the optimal selection of the interferometer system configuration in terms of the acceptable frequency detection error, operating bandwidth, or even the topological distribution of the ports.

**Keywords:** microwave correlator, phase shifter, Six-Port, directional coupler, microwave frequency discriminator

## 1. INTRODUCTION

Microwave frequency discriminators (MFD) are systems used to develop information on the frequency of the received signal [1-3,5]. This task can be implemented in systems with frequency conversion, systems with microwave resonators or even band-pass filters. Very good results of broadband measurement of the frequency of microwave signals are achieved in the MFD, where interferometric properties are used[1-3]. Outputs slow-changing signals are obtained whose value depends on the frequency of the input signal. The overall structure of the discriminator consists of the element generating two signals, whose phase difference is proportional to the frequency of the input signal. One of the possible implementations of this task consists of a signal distribution element and two transmission lines of different lengths [4]. These components create the so-called system for developing a proportional phase difference. Another essential element of MFD is the microwave phase detector system (MPhD), which is designed to generate slow-changing signals proportional to the phase difference of the two signals obtained at an earlier stage. To increase the frequency discrimination resolution, lines with a larger length can be used. However, it should be noticed that such a procedure narrows the measurement unambiguity band [5,6]. Therefore, for correct operation, several frequency discriminators with different line length are used. With the increase in the number of MFD with this appropriately selected difference, the band for unambiguous frequency detection or the accuracy of its determination increases. Another limitation in the use of longer lines is the duration of the pulse which frequency we want to measure. The pulse duration must allow interference signal with signal with a proportional phase difference. The speed of operation of these systems means that very often they are an essential element of IFM (Instantaneous Frequency Measurement) receivers [7].

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## 2. ANALYSIS OF MICROWAVE FREQUENCY DISCRIMINATOR

### 2.1 Principle of operation

The classic interferometric MFD model consists of two basic functional blocks. The system generating two signals  $u_{w1}$  and  $u_{w2}$  which phase difference is proportional to the frequency of the input signal and the microwave phase detector (MPhD) system.

$$u_{we} = U_{we} \cdot e^{j(2\pi ft + \varphi_w)} \quad (1)$$

The first of these systems consists of the Wilkinson DM power divider, which provides equal distribution of the input signal energy (1) and the SL delay line – figure 1.

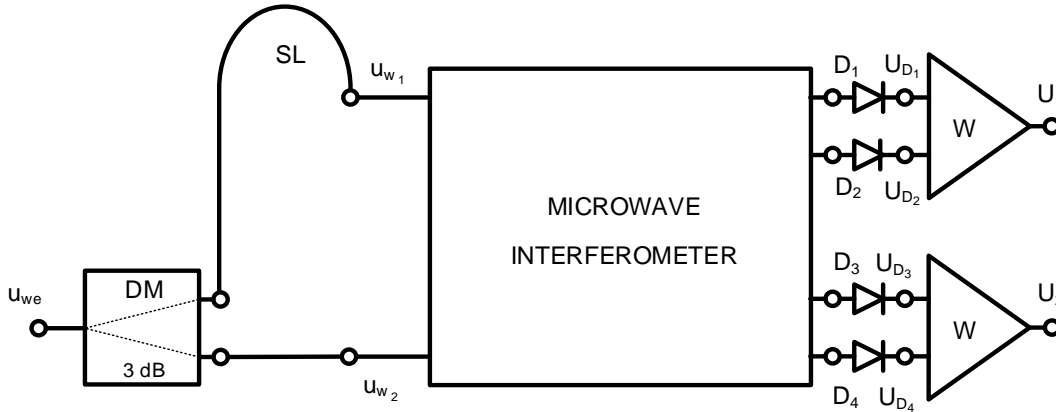


Figure 1 Block diagram of an interferometric microwave frequency discriminator.

After that, these signals are fed to the microwave phase discriminator system, which can be made in the form of a microwave interferometer. This system implements the appropriate division of the  $u_{w1}$  and  $u_{w2}$  signals, which in the next stage are summed vectorly. Afterwards, develops four signals whose slowly varying amplitude - after quadrature detection, depends on the frequency of the input signal.

### 2.2 Components

As mentioned earlier, the classic microwave frequency detector usually includes such elements as: quadrature directional couplers, power dividers and delay lines – figure 2.

In the analyzed case, a structure was tested that includes:

- delay line, which is responsible for the phase delay of the signal:

$$S_{SL} = e^{-j\gamma} \quad (2)$$

where:  $\gamma = \frac{2\pi}{c} \cdot f \cdot l_{SL}$ ,  $l_{SL}$  – length of SL line

- Wilkinson power divider:

$$S_D = \frac{2}{3 \cos(\theta(f)) + j 2\sqrt{2} \sin(\theta(f))} \quad (3)$$

- directional coupler with transmittance to direct port:

$$S_B = \frac{\sqrt{1-k^2}}{\sqrt{1-k^2} \cos(\theta(f)) + j \sin(\theta(f))} \quad (4)$$

and to coupled port:

$$S_S = \frac{jk \sin(\theta(f))}{\sqrt{1-k^2} \cos(\theta(f)) + j \sin(\theta(f))} \quad (5)$$

where:  $\theta(f)$  – electrical length of the coupling area;

$k$  – coupling factor.

In general considerations it is accepted that:

$$\theta = \frac{\pi f}{2 f_0} \quad (6)$$

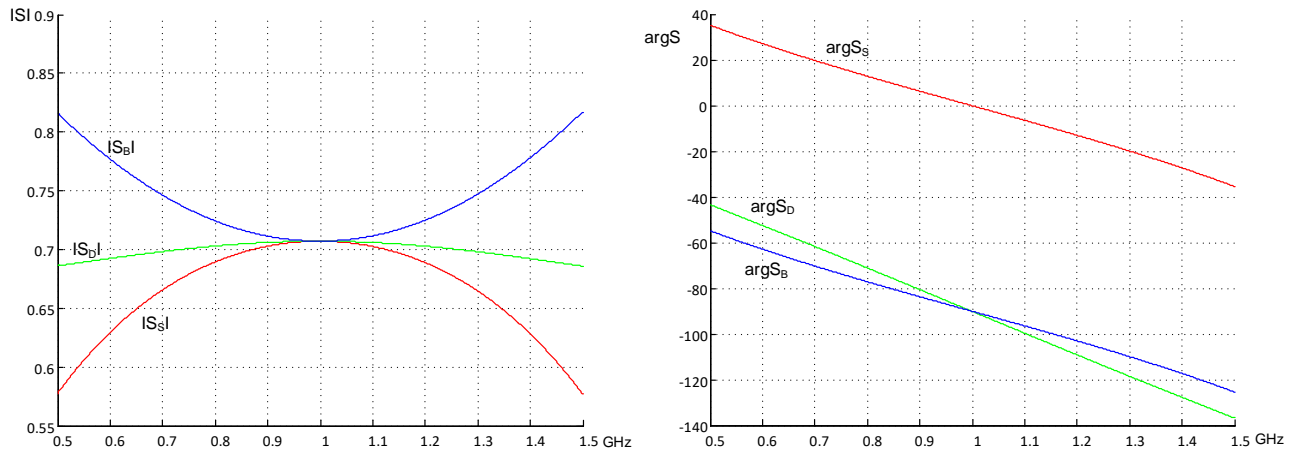


Figure 2 Transmission parameters of the power divider and quadrature coupler used in the discriminator.

### 2.3 MFD based on Butler Matrix

In the paper it was decided to analyze the MFD system in which the  $4 \times 4$  Butler Matrix was used as the interferometer of the microwave phase detector – figure 3.

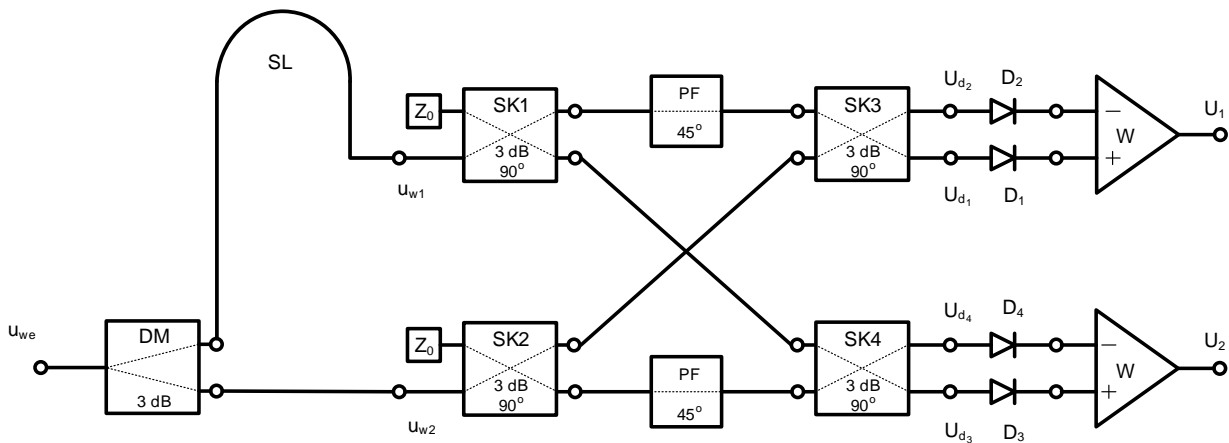


Figure 3 Block diagram of the microwave frequency discriminator using the Butler  $4 \times 4$  matrix.

It is worth to noticed that such a system is built of four SK quadrature couplers and two broadband phase shifters PF. These shifts provide a relative phase shift of  $45^\circ$ :

$$S_{PF} = e^{-j\frac{\pi}{4}} \quad (7)$$

With this information, we can determine the transmittance between the input and output ports of the interferometer, and then determine the input voltage of each microwave signal detector:

$$U_{dj} = \sum_{i=1}^2 T_{ji} u_{wi} \quad (8)$$

$T_{ji}$  transmittance is a complex function of parameters of system elements - their transmittance, isolation and reflection from the port. A full analysis of the impact of these components on the interferometer parameters can be visualized using a full flow graph. A simplified signal flow graph is presented in figure 4 and figure 5. Simplifications were acceptable because some assumptions were made:

- for the coupler: matched inputs and outputs, perfect directivity, full symmetry;
- for power divider: matched inputs and outputs, full symmetry, perfect output separation;
- for phase shifter: matched inputs and outputs, fixed relative shift phase value.

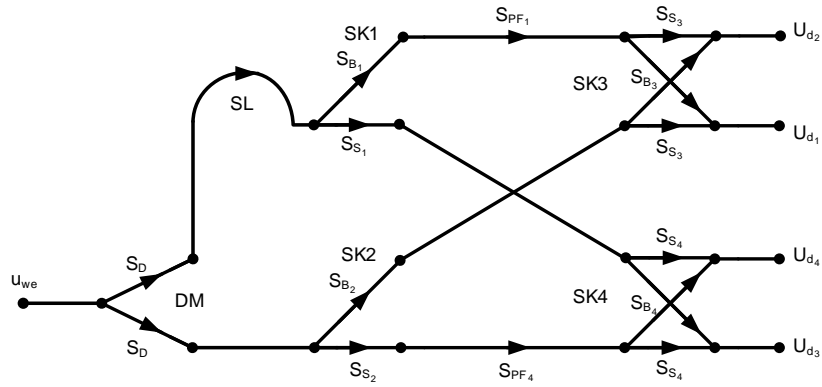


Figure 4 Simplified signals flow graph for connecting SL line to SK1 coupler.

Figure 4 shows a simplified graph of signal flow when the SL line is connected between the power divider and SK1 coupler, while figure 5 shows the situation in which this line is connected between the power divider and SK2 coupler.

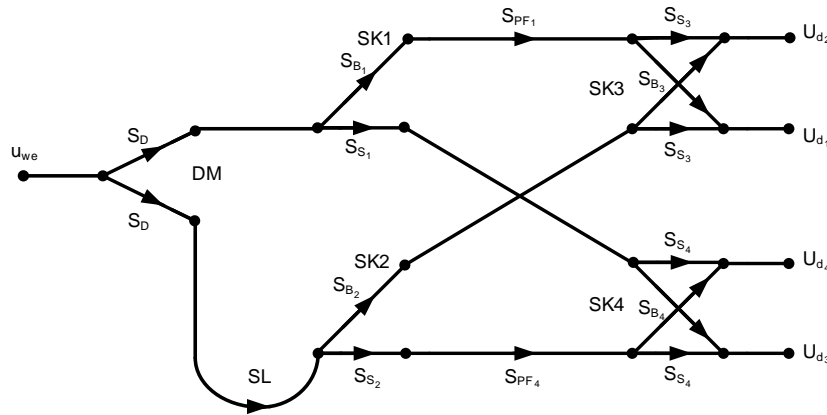


Figure 5 Simplified signals flow graph for connecting SL line to SK2 coupler.

Signals at the MFD output, assuming a quadratic characteristic of detection diodes and equal detection efficiency, as slow-changing voltage signals are determined by the relationships:

$$U_1 = U_{D_1} - U_{D_2} = 0,5 \cdot k_d \cdot U_{we}^2 \cdot \sin(\theta - \frac{1}{4}\pi) \quad (9)$$

$$U_2 = U_{D_3} - U_{D_4} = 0,5 \cdot k_d \cdot U_{we}^2 \cdot \cos(\theta - \frac{1}{4}\pi) \quad (10)$$

where:  $\theta = \frac{2\pi}{c} \cdot f \cdot l_{SL}$ ,  $l_{SL}$  – length of SL line;

Submission of  $U_1$  and  $U_2$  output signals allows the elimination of amplitudes and obtaining the tangent of the angle by which the signal will be delayed passing through the SL line. Therefore, to determine the frequency of the input signal, the equation should be used:

$$f = \frac{(\arctan(\frac{U_1}{U_2}) + \frac{\pi}{4}) \cdot c}{2 \cdot \pi \cdot l_{SL}} \quad (11)$$

### 3. RESULTS

The calculations were performed in the MATLAB computer program, which enables simulations for various values of system parameters. For better presentation of systems parameters the values of frequency detection deviations are presented in the charts for the value  $f_0 = 1\text{GHz}$

Figure 6 shows the plot of frequency detection deviation. Calculations were made for the coupling factor of couplers equal to -3dB and for both ways of implementation a line generating a proportional phase difference.

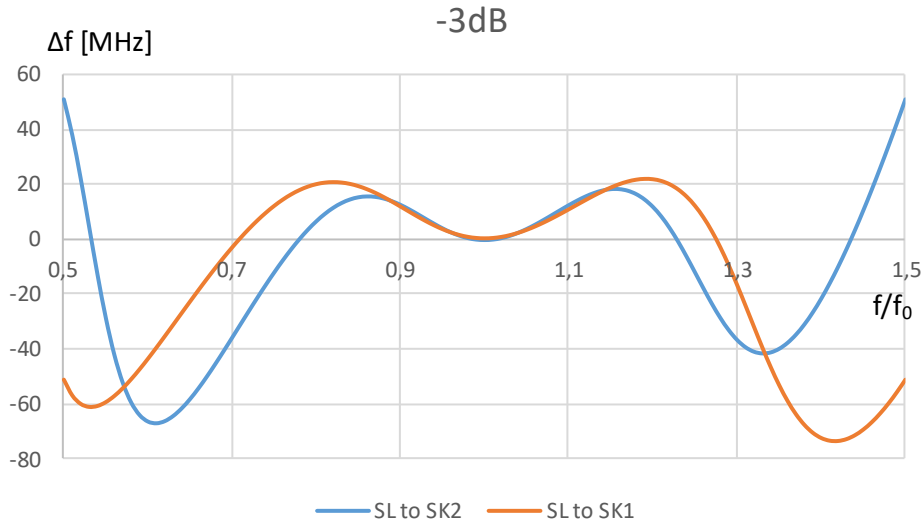


Figure 6 The graph of the MFD frequency detection deviation for the coupling factor of the couplers at the level of -3 dB and with different ways of connecting the SL line.

It turns out that in the "SL to SK2" configuration, we can obtain a working band  $(0.74 \div 1.26) f/f_0$  for the assumed deviation of  $\pm 18.2$  MHz, while for the "SL to SK1" configuration, we obtain a narrower band  $(0.86 \div 1.15) f/f_0$ . However, if only a deviation of  $\pm 22$  MHz is allowed, the situation reverses and a wider band  $(0.65 \div 1.3) f/f_0$  is obtained for the "SL to SK1" system compared to  $(0.73 \div 1.27) f/f_0$  for the "SL to SK2" system.

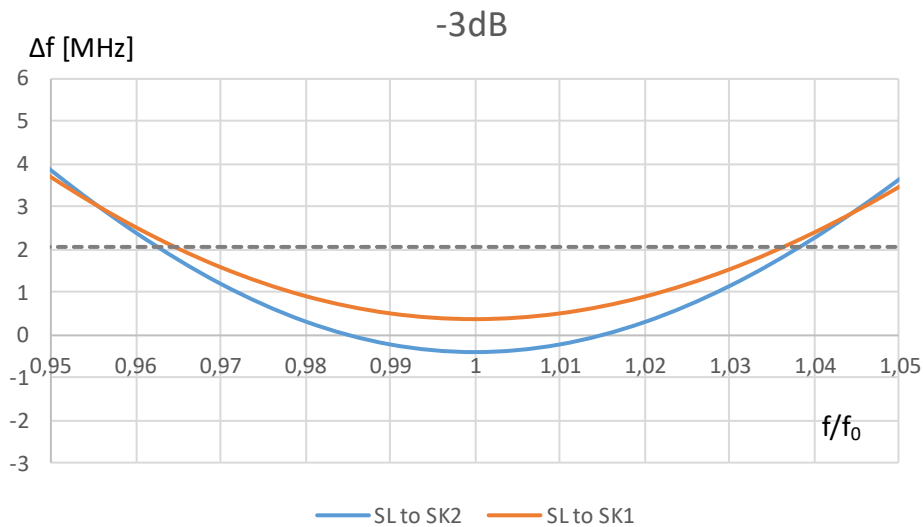


Figure 7 The graph of the MDCz frequency detection deviation for the coupling factor of the couplers at the level of -3 dB and with different ways of connecting the SL line for a narrowed frequency band.

However, if only the frequency band is narrowed to the range  $f/f_0 \pm 0.05 \cdot f/f_0$  and the appropriate calculations are made, a frequency detection error deviation not greater than  $\pm 1.67$  MHz is possible to achieve for the system when the SL line is connected to SK1 and detection error can be obtained not greater than  $\pm 2.43$  MHz for the other configuration – figure 7.

Calculations were also made for different ways of implementing the SL line and various coupling factors SK. Figure 8 shows the results for the system in which this line was connected between the power divider and the SK2 interferometer coupler.

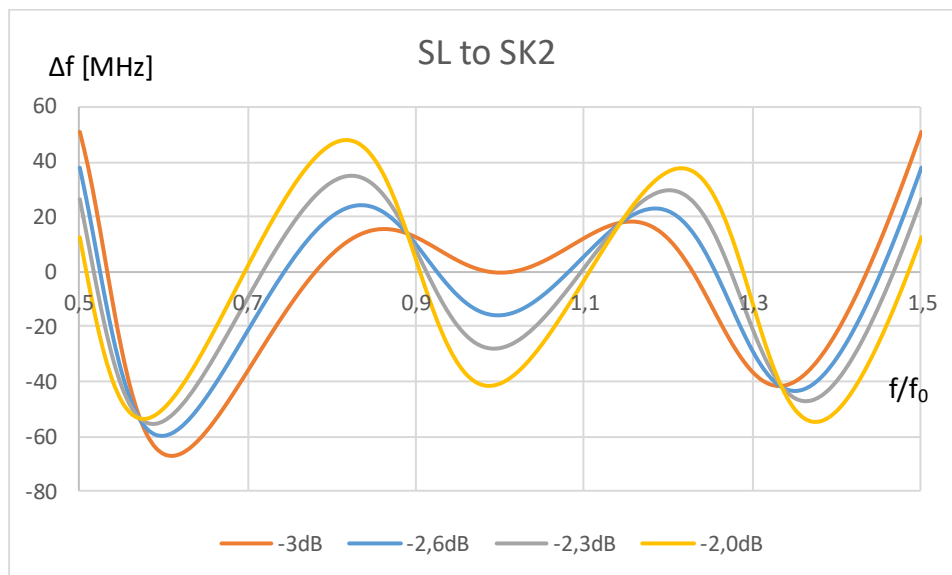


Figure 8 The graph of the MDCz frequency detection deviation for various coupling factors of the couplers and the SL line connected to SK2.

For the -3dB coupling factor in the tested frequency band ( $0.5 \div 1.5$ )  $f/f_0$  the frequency detection deviation is in the range from -67MHz to +51MHz. As the ratio increases from -2.6dB through -2.3dB to -2dB, the deviation range changes in the

ranges from -59MHz to + 38MHz, from -55.6MHz to + 34.6MHz, from -54.7MHz to + 47,8MHz. For the situation when the allowable deviation of  $\pm 18.2$ MHz for the frequency close to the center frequency of the system is limited, it turns out that this condition will be met for a system with -3dB coupling in the frequency band  $(0.74 \div 1, 26) f/f_0$  and with -2.6dB coupling in the band  $(0.88 \div 1.14) f/f_0$ . On the other hand, for larger couplings, this deviation is greater than the specified range.

Figure 9 shows the results of calculations for the case of placing the SL line between the power divider and the SK1 interferometer coupler.

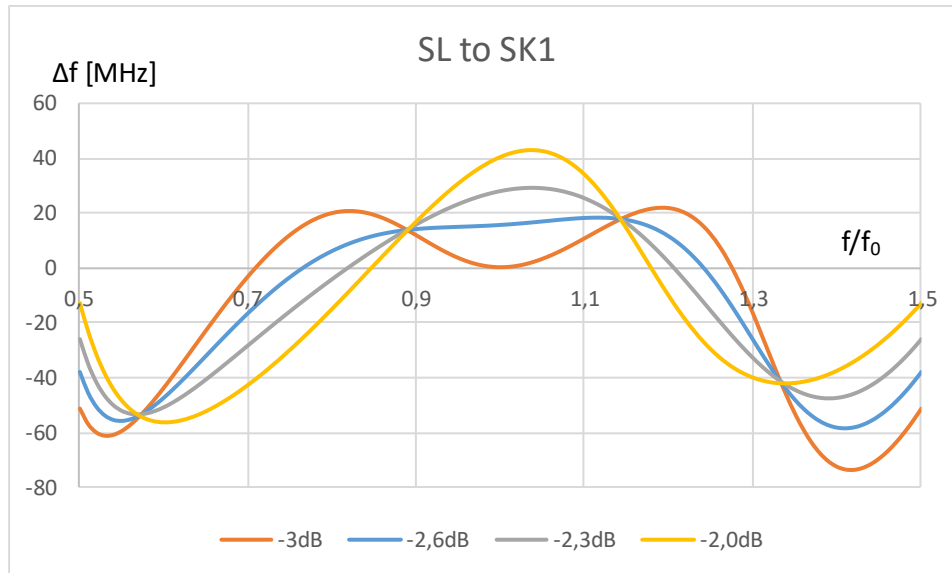


Figure 9 The graph of the MDCz frequency detection deviation for various coupling factors of the couplers and the SL line connected to SK1.

For the -3dB coupling factor in the tested frequency band  $(0.5 \div 1.5) f/f_0$  the frequency detection deviation is in the range from -73.2MHz to + 22MHz. As the ratio increases from -2.6dB through -2.3dB to -2dB, the deviation range changes in the ranges from -58.4MHz to + 18.4MHz, respectively from -53.6MHz to + 29.2MHz, from -55.9MHz to + 43.2MHz. For the situation when the allowable deviation is limited, e.g. to the value of  $\pm 22$ MHz for frequencies near the center frequency of the system, it turns out that this condition will be met for a system with -3dB coupling in the frequency band  $(0.65 \div 1.31) f/f_0$  and with -2.6dB coupling in the band  $(0.68 \div 1.29) f/f_0$ . On the other hand, for larger couplings, this deviation is greater than the specified range.

Figure 6 shows the relation of the MFD detection deviation to the SL line length. Three values were adopted: wavelength of center frequency, its fourfold and its sixteen fold. In order to enable comparison of individual solutions, appropriate conversions were carried out, which eliminated the ambiguity of frequency discrimination resulting from the accepted multiple of SL line length.

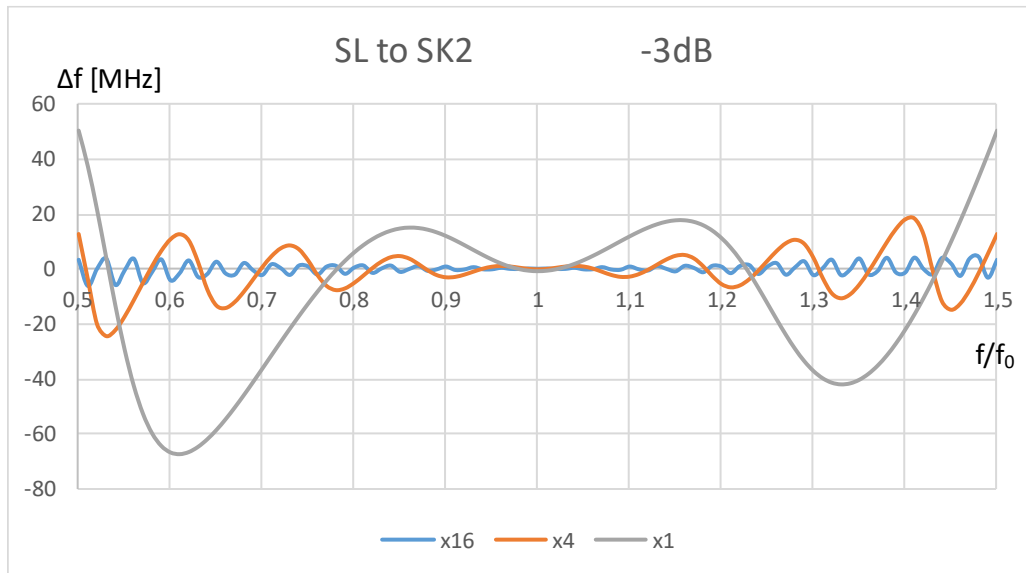


Figure 10 The graph of the MFD frequency detection deviation at the coupling factors of the couplers at the level of -3dB and for three different lengths of the SL line connected to SK2.

For a system with SL line length equal to wavelength for  $f_0 = 1\text{GHz}$ , the frequency detection deviation range was obtained within the range from -67MHz to + 51MHz. Subsequently, the deviation range was from -24.3MHz to + 18.3MHz for line length 4 x SL and from -6.2MHz to + 4.2MHz for line length 16 x SL.

#### 4. CONCLUSION

In the paper it was pointed out that the detection deviation value of the microwave frequency detector depends on the topological distribution of components - in the analyzed case the influence of the connecting place of the line generating a proportional phase difference (SL line) was checked. For the same coupling factors of -3dB couplers, a deviation of not more than  $\pm 18.2\text{MHz}$  was achieved for the system with the line connected to SK2 in the  $0.52f_0$  band, while for the other configuration of the simulated discriminator this band was  $0.29f_0$ . But for a larger allowable deviation of  $\pm 22\text{MHz}$ , the situation is reversed and a wider bandwidth at  $0.55f_0$  is characterized by a system with a line connected to SK1, while for the other system the bandwidth is  $0.54f_0$ .

The property of MFD discrimination also depends on the value of the coupling factor of the couplers used. In the entire tested frequency band, for the system with the SL line connected to SK2, the smallest range of frequency detection deviations equal to  $\pm 90.2\text{MHz}$  is obtained for a coupling of -2.3dB. However, for a system with an SL line connected to SK1, the smallest range of frequency detection deviations equal to  $\pm 76.8\text{MHz}$  is obtained for a coupling of -2.6dB.

The paper presents a proposal of an analog frequency measurement system. It is also worth to notice that extending the SL line significantly improves the ability to detect frequencies from the deviation range of 118MHz for the variant with SL line length equal to wavelength  $f_0$  to the range of deviation equal to 10.4MHz for sixteen times longer, i.e. we note an improvement of more than eleven. For long SL lines there is a problem of ambiguity when measuring the frequency in a wide band. That is why in further works it is proposed to use digital multi-channel systems to eliminate ambiguity. This will allow to use the MDF system proposed in the work to build frequency measurement systems in a very wide operating band and with high accuracy of frequency determination.

In connection with the abovementioned relationships, MFD parameters should be determined in order to make the optimal selection of the detector system configuration in terms of the allowable error of frequency detection, operating bandwidth or topological distribution of its individual components.



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