Spatial filtering of Structured Light

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Abstract: We present an approach to filter amplitude noise from arbitrary beam profiles. This work forms a tutorial on Fourier optics by enhancing the textbook case of Gaussian beams to more interesting cases of structured light. © 2021 The Author(s)

1.   Introduction

The technique of simple pinholes in spatial filtering predates the invention of the laser [1] and is still commonly used today in the "textbook" removal of high-frequency amplitude noise from Gaussian beams. In the context of structured light, we ask: how can we remove additive noise from arbitrary incoming light to achieve a desired, noise-free field? Relying heavily on concepts from Fourier optics, we present a tutorial-style method on spatially removing unwanted noise from structured light [2].

2.   Fourier optics

The Fourier transform, denoted $\mathscr{F}$, and its inverse $\mathscr{F}^{-1}$ are well-known integral transforms, relating a field $u(\mathbf{x})$ to its frequency spectrum $U(\mathbf{k})$. The field U reveals which spatial frequencies are present in the original field u. If a lens is placed a focal length f in front of u, then the field U is formed a distance f behind the lens, at the focal plane. That is, the lens optically performs the Fourier transform on u and so the focal plane is often referred to as the "Fourier" plane.
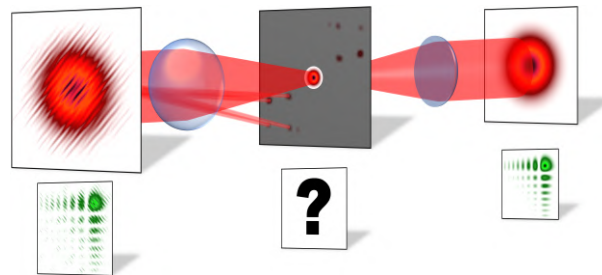
3.   Textbook spatial filtering

Fig. 1. "Textbook" spatial filtering (first row): high frequency amplitude noise in the image plane is spatially separated from the signal beam in the Fourier plane using a lens. A pinhole allows the signal to be passed through, blocking the noise. Another lens is used to return the beam to the image plane. What mask is necessary to filter a structured light beam (second row) and how can it be implemented?

Figure 1 illustrates the process of spatial filtering using a lens and mask. High frequency noise can be separated from a Gaussian beam (top row) by Fourier transforming, as the frequencies constituting the noise are higher than those of the beam. The beam and noise will thus be distinct at the Fourier (focal) plane. A binary mask blocks the noise but transmits the beam, with the process being completed by a second lens which transforms back to the image plane. The process is more complicated for structured light (second row). We must answer the question: what mask must be constructed?

4.   Generalised spatial filtering

To construct a binary mask for arbitrary beam profiles, we note that the amplitude of the beam is larger than that of the noise at the Fourier plane. This allows us to construct a binary mask M, as seen in Figure 2, for a noisy field

$u(\mathbf{x})$ at the Fourier plane by

$$M = \begin{cases} 1, & \text{if } |\mathscr{F}\{u(\mathbf{x})\}| \geq \mathscr{A}\exp(-t^2) \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where $\mathscr{A}$ is the maximum signal amplitude at the Fourier plane and t is a dimensionless value which we call the mask width parameter. Experimental examples for Bessel-Gaussian (BG) and Hermite-Gaussian (HG) beams
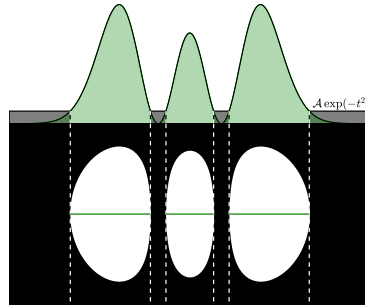

Fig. 2. The binary mask is constructed by thresholding the initial field according to Eq. (1), shown here for an Hermite-Gaussian mode along the center row. The parameter t determines the width of the mask elements.

using this technique are shown in Figure 3 for low-frequency Gaussian and high-frequency noise, respectively. However, this generalised method is not successful if the constituent frequencies of the noise lie within the Fourier transform of the 'clean' field..
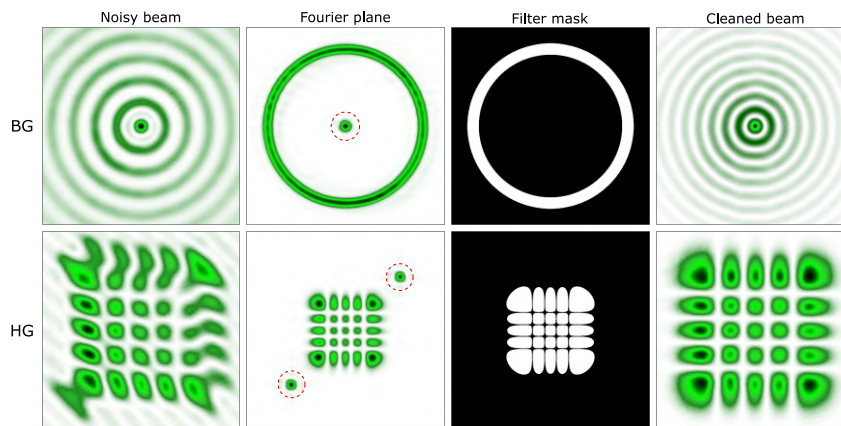

Fig. 3. Experimental spatial filtering examples of two structured light fields: Bessel-Gaussian and Hermite-Gaussian modes where low-frequency (Gaussian) and high-frequency amplitude noise was added, respectively. Dashed circles in the second column indicate the location of the noise in the Fourier plane.

5. Discussion and conclusion

We have married computational and experimental tools almost seamlessly. The binary masks are "pictures" which can be easily displayed digitally in the laboratory using liquid crystal displays or digital micromirror devices. The experimental is thus only digital and can be incorporated quickly into a teaching course as a computational exercise. To assist in this, we have provided all our code for generating the computer holograms [3].

References

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3. https://github.com/JPinnell/Spatial-filtering-of-structured-light.