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**Abstract.** This study proposes an alternative and simple method for measuring full-field refractive index. This method is based on the phase-shifting technique with a modulated electro-optical (EO) modulator and the phenomenon of total internal reflection. To this purpose, a linear polarized light is expanded and incident on the interface between the prism and the tested specimen, and the reflected light passes through an analyzer for interference. The phase difference between the *s*- and *p*-polarized light is sensitive to the refractive index of the tested specimen when the total internal reflection appears on this interface. Based on this effect, the resulting phase differences make it possible to analyze the refractive index of the tested specimen through a phase-shifting technique with a modulated EO modulator. The feasibility of this method was verified by experiment, and the measurement resolution can reach a value of refractive index unit of at least  $3.552 \times 10^{-4}$ . This method has advantages of simple installation, ease of operation, and fast measurement. © The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: [10.1117/1.OE.54.9.094101](https://doi.org/10.1117/1.OE.54.9.094101)]

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## 1 Introduction

The measurement of full-field refractive index exhibits its significance and has many important applications in many specific fields, including the inspection of optical material and thin-film characteristic as well as research in biology, biochemistry, and medicine. Currently, some methods have been proposed to measure full-field refractive index, such as the interferometric methods,<sup>1–6</sup> digital holographic microscopy,<sup>7</sup> and ellipsometric methods.<sup>8</sup> In the interferometric methods, the phase difference is relative to the refractive index. To obtain the phase difference distribution to measure full-field refractive index, the phase-shifting technique is an important and effective method that features simple structures, easy operation, and rapid measurement. Generally, the phase-shifting methods introduce successive phase steps into interferometric signals with various phase-shifting strategies, such as piezoelectric transducer,<sup>1,2</sup> holographic grating,<sup>3</sup> and tunable wavelength.<sup>4</sup> Aforementioned methods could suffer from mechanical perturbations, and their costs are high. In addition, to reveal the phase distribution, techniques of three-step and four-step phase-shifting methods are commonly applied. However, the introduced values of phase-shifting must be specified. Therefore, this study proposes an alternative and simple method based on the Carré phase-shifting technique with modulated electro-optical (EO) modulator for measuring full-field refractive index. A linear polarized light is expanded and incident on the interface between the prism and the tested specimen. When the total internal reflection appears on this interface, a significant phase difference between the *s*- and *p*-polarized light is

introduced. This phase difference is relative to the refractive index of the tested specimen, and can be precisely measured with phase-shifting technique with modulated EO modulator. Accordingly, the full-field refractive index of the tested specimen can be estimated. To prove the feasibility of the proposed method, the tested specimen combining with pure water and castor oil is measured. The experimental data correspond well with the theoretical values. Due to the introduction of EO modulation and Carré phase-shifting techniques, the proposed measurement method can avoid mechanical perturbation and has merits of simple installation, ease of operation, and fast measurement.

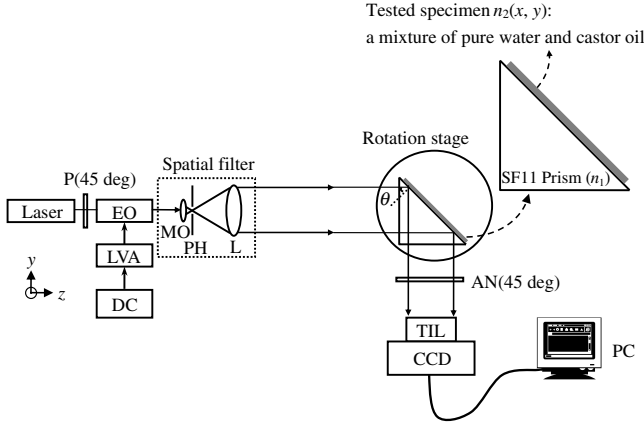
## 2 Principles

Figure 1 shows the optical configuration of the proposed method. For convenience, the *z*-axis is set in the direction of light propagation, and the *x*-axis is set perpendicular to the plane of the paper. Linear polarized laser light with the polarization state at 45 deg with respect to the *x*-axis passes through an EO modulator with the optic axis at 0 deg with respect to the *x*-axis. Using a DC power supply (DC) and a linear-voltage amplifier (LVA) to drive the EO modulator, the Jones vector of the electric field of the resulted laser light  $E_{in}$  can be written as

$$E_{in} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{\frac{i\Gamma}{2}} \\ e^{-\frac{i\Gamma}{2}} \end{pmatrix} e^{i\omega_0 t}, \quad (1)$$

where  $\omega_0$  is the light frequency, and  $\Gamma$  denotes the phase retardation between the *s*- and *p*-polarizations, which can be written as<sup>9</sup>

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**Fig. 1** Schematic diagram for measuring the full-field refractive index, P: polarizer; EO: electro-optic modulator; LVA: linear-voltage amplifier; DC: DC power supply; MO: microscopic objective; PH: pinhole; L: collimating lens; AN: analyzer; TIL: telecentric imaging lens; CCD: CMOS camera; PC: personal computer.

$$\Gamma = \pi \frac{V_z}{V_\pi}, \quad (2)$$

where  $V_\pi$  is the half-wave voltage of the EO modulator, and  $V_z$  is an external voltage applied to the EO modulator. Then, the light beam is expanded and collimated by a spatial filter to form a plane wave, and incident on the undersurface of a prism at an angle  $\theta$ . The refractive indices of the prism and the tested specimen are  $n_1$  and  $n_2$ , respectively. When the incident angle is larger than the critical angle  $\theta_c = \sin^{-1}(n_2/n_1)$ , the total internal reflection appears. The reflected light passes through an analyzer with the transmission axis at 45 deg with respect to the  $x$ -axis, and finally reaches a CMOS camera (CCD) by a telecentric imaging lens (TIL). The Jones vector of the light at the CMOS camera can be written as

$$E(x, y) = \frac{1}{2\sqrt{2}} \left[ |r_p(x, y)| e^{\frac{\pi}{2} + i\phi_p(x, y)} + |r_s(x, y)| e^{-\frac{\pi}{2} + i\phi_s(x, y)} \right] \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (3)$$

where  $r_p(x, y)$ ,  $r_s(x, y)$ ,  $\phi_p(x, y)$ , and  $\phi_s(x, y)$  denote the reflection coefficients and phase differences of  $p$ - and  $s$ -polarizations, respectively. These parameters can be written as<sup>10</sup>

$$r_p(x, y) = \frac{n^2(x, y) \cos \theta - i\sqrt{\sin^2 \theta - n^2(x, y)}}{n^2(x, y) \cos \theta + i\sqrt{\sin^2 \theta - n^2(x, y)}} = |r_p(x, y)| e^{i\phi_p(x, y)}, \quad (4)$$

$$r_s(x, y) = \frac{\cos \theta - i\sqrt{\sin^2 \theta - n^2(x, y)}}{\cos \theta + i\sqrt{\sin^2 \theta - n^2(x, y)}} = |r_s(x, y)| e^{i\phi_s(x, y)}, \quad (5)$$

where  $n = n_2/n_1$  is the relative refractive index. Therefore, the intensity measured at the CMOS camera can be expressed as

$$I(x, y) = |E(x, y)|^2 = \frac{1}{4} \{ |r_p(x, y)|^2 + |r_s(x, y)|^2 + 2|r_p(x, y)||r_s(x, y)| \cos[\Gamma + \phi(x, y)] \}, \quad (6)$$

where  $\phi(x, y)$  is the phase difference between the  $p$ - and  $s$ -polarized light from the reflection at the boundary surface under the conditions of total internal reflection, and it can be expressed as

$$\phi(x, y) = \phi_p(x, y) - \phi_s(x, y) = \arg[r_p(x, y)] - \arg[r_s(x, y)]. \quad (7)$$

Substituting Eqs. (4) and (5) into Eq. (7), the relationship of the phase difference  $\phi(x, y)$  and the relative refractive index  $n(x, y)$  can be expressed as

$$\phi(x, y) = -2 \tan^{-1} \left[ \frac{\sqrt{\sin^2 \theta - n^2(x, y)}}{\tan \theta \sin \theta} \right], \quad (8)$$

Eq. (8) can also be rewritten as

$$n_2(x, y) = n_1 \sin \theta \left\{ 1 - \tan^2 \left[ \frac{\phi(x, y)}{2} \right] \cdot \tan^2 \theta \right\}^{1/2}. \quad (9)$$

According to Eq. (9), the refractive index  $n_2(x, y)$  of the tested specimen is the function of the phase difference  $\phi(x, y)$ . Hence, the refractive index  $n_2(x, y)$  of the tested specimen can be obtained by an accurate measurement of the phase difference  $\phi(x, y)$ .

To measure the phase difference  $\phi(x, y)$ , the phase-shifting technique with modulated EO modulator is introduced. From Eq. (6), the different voltages of  $-(3/2)V_1$ ,  $-(1/2)V_1$ ,  $(1/2)V_1$ , and  $(3/2)V_1$  are sequentially entered into the EO modulator to obtain different phase shifting. Then, the four sets of interferometric signals can be obtained and can be respectively expressed as

$$I_1(x, y) = \frac{1}{4} \left\{ |r_p(x, y)|^2 + |r_s(x, y)|^2 + 2|r_p(x, y)||r_s(x, y)| \cos \left[ \phi(x, y) - \frac{3\pi V_1}{2V_\pi} \right] \right\}, \quad (10)$$

$$I_2(x, y) = \frac{1}{4} \left\{ |r_p(x, y)|^2 + |r_s(x, y)|^2 + 2|r_p(x, y)||r_s(x, y)| \cos \left[ \phi(x, y) - \frac{\pi V_1}{2V_\pi} \right] \right\}, \quad (11)$$

$$I_3(x, y) = \frac{1}{4} \left\{ |r_p(x, y)|^2 + |r_s(x, y)|^2 + 2|r_p(x, y)||r_s(x, y)| \cos \left[ \phi(x, y) + \frac{\pi V_1}{2V_\pi} \right] \right\}, \quad (12)$$

$$I_4(x, y) = \frac{1}{4} \left\{ |r_p(x, y)|^2 + |r_s(x, y)|^2 + 2|r_p(x, y)||r_s(x, y)| \cos \left[ \phi(x, y) + \frac{3\pi V_1}{2V_\pi} \right] \right\}. \quad (13)$$

By using the Carré algorithm,<sup>11,12</sup> the phase difference  $\phi(x, y)$  can be calculated as

$$\phi(x, y) = \tan^{-1} \frac{\sqrt{\{[I_1(x, y) - I_4(x, y)] + [I_2(x, y) - I_3(x, y)]\} \{3[I_2(x, y) - I_3(x, y)] - [I_1(x, y) - I_4(x, y)]\}}}{[I_2(x, y) + I_3(x, y)] - [I_1(x, y) + I_4(x, y)]}. \quad (14)$$

Therefore, the phase difference  $\phi(x, y)$  can be determined with accurately measured intensities  $I_1(x, y) \sim I_4(x, y)$ . According to Eq. (9), the full-field refractive index  $n_2(x, y)$  of the tested specimen then can be obtained.

### 3 Experimental Results and Discussions

To prove the feasibility of the proposed method, the tested specimen, a mixture of pure water ( $n_w = 1.331$  at 632.8 nm) and castor oil ( $n_o = 1.480$  at 632.8 nm) with a refractive index distribution  $n_2(x, y)$ , was measured at room temperature of 25°C. A He-Ne laser with a wavelength of 632.8 nm that was expanded with a diameter of 4.5 mm was used as the test light source. The EO modulator (Mode 4002, New Focus) is driven by a DC power supply (GPD3303, GWINSTEK) and an LVA (Mode 3211, New Focus). The different phase retardations of EO modulator were modulated by sequentially supplying different voltage values of -120 V, -40 V, 40 V, and 120 V. A high-resolution motorized rotation stage (Model SGSP-60-WPQ, Sigma Koki, Inc.) with an angular resolution of 0.005 deg was used to mount and rotate the tested apparatus. The tested apparatus consisted of an SF11 right-angle prism ( $n_1 = 1.778$ ) with the tested specimen on its base. To achieve high sensitivity, the incident angle  $\theta$  of light at the base of the prism was set to 57 deg. A telecentric lens (Silver Series Telecentric Lens, Edmund Optics Inc.) with a primary magnification of 0.25× and a working distance of 160 mm was mounted on an 8-bit gray level CMOS camera (XCD-U100CR, Sony Electronics Inc.) for imaging the interferometric signals. The pixels of the CMOS camera were  $1600 \times 1200$  with a cell size of  $4.4 \mu\text{m} \times 4.4 \mu\text{m}$  (sensor format 1/1.8-type). A personal computer and the MATLAB® software were used to analyze the captured images. The experimental results are shown in Figs. 2–5. Figure 2 shows the four interferometric images of various phase distributions. Figures 3 and 4 show the measured phase difference  $\phi(x, y)$  distribution and refractive index  $n_2(x, y)$  distribution, respectively. Figure 5 shows the distribution of refractive index along the  $x$ -axis at  $y = 3.5$  mm, in which the refractive indices of 1.336 and 1.482 are corresponding to pure water and castor oil, respectively. The measured results are in good agreement with the reference values, demonstrating the capability of the proposed method.

Considering the errors of phase and incident angle, the refractive index measurement resolution  $\Delta n_2$  can be estimated. According to Eq. (9), it can be expressed as

$$\begin{aligned} \Delta n_2 &= \left| \frac{\partial n_2}{\partial \phi} \right| \Delta \phi_{\text{err}} + \left| \frac{\partial n_2}{\partial \theta} \right| \times \Delta \theta_{\text{err}} \\ &= \left| \frac{-B \sec^2\left(\frac{\phi}{2}\right) \tan \theta}{2A} \right| \times \Delta \phi_{\text{err}} \\ &\quad + \left| A n_1 \cos \theta - \frac{B \tan\left(\frac{\phi}{2}\right) \sec^2 \theta}{A} \right| \times \Delta \theta_{\text{err}}, \end{aligned} \quad (15)$$

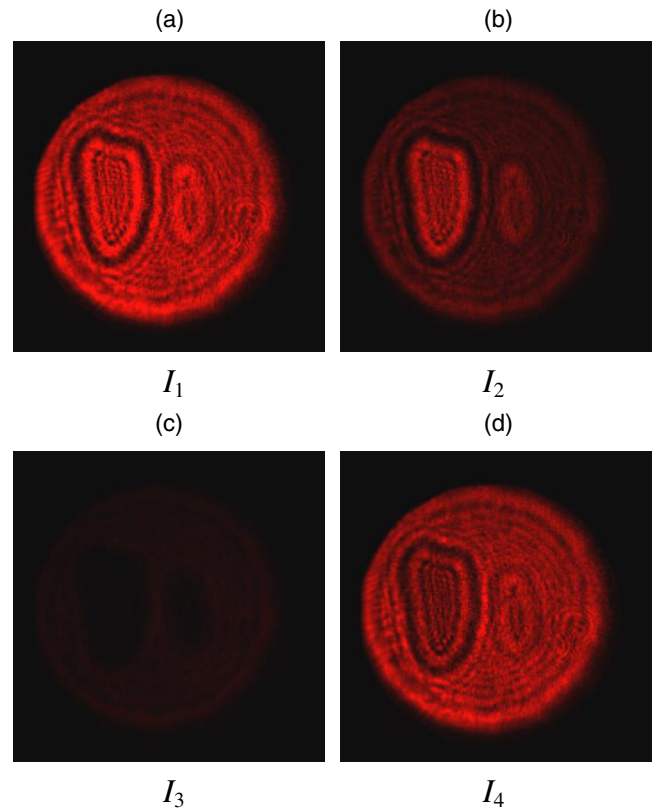
where

$$A = \sqrt{1 - \tan^2\left(\frac{\phi}{2}\right) \tan^2 \theta}, \quad (16)$$

and

$$B = n_1 \tan\left(\frac{\phi}{2}\right) \sin \theta \tan \theta, \quad (17)$$

where  $|\Delta \phi|$  denotes the total phase error in the experiment,  $|\Delta \theta|$  represents the error of the incident angle at the base of the right-angle prism. The  $\Delta \phi$  can be estimated by considering the CMOS camera resolved-phase error, the polarization-mixing error, and the phase-shifting modulated error. The theoretical resolution of gray-level interferometric signals is about  $\Delta I \cong 1/256 = 3.9 \times 10^{-3}$  with an 8-bit CMOS camera. Therefore, the CMOS camera resolved-phase error was 0.0056 deg by substituting the resolution of interferometric signal  $\Delta I$  into Eqs. (10) to (14). The extinction ratio of the polarizer (Newport Inc.) is  $1 \times 10^{-3}$ , so the polarization-mixing error was estimated with value of 0.0072 deg. In this experimental setup, a DC power supply with the voltage resolution of 0.03% was used to drive the EO modulator, the phase-modulated error  $\Delta \Gamma$  was about



**Fig. 2** Four interferometric images of various phase distributions with EO modulation driving on (a) -120 V, (b) -40 V, (c) 40 V, and (d) 120 V.

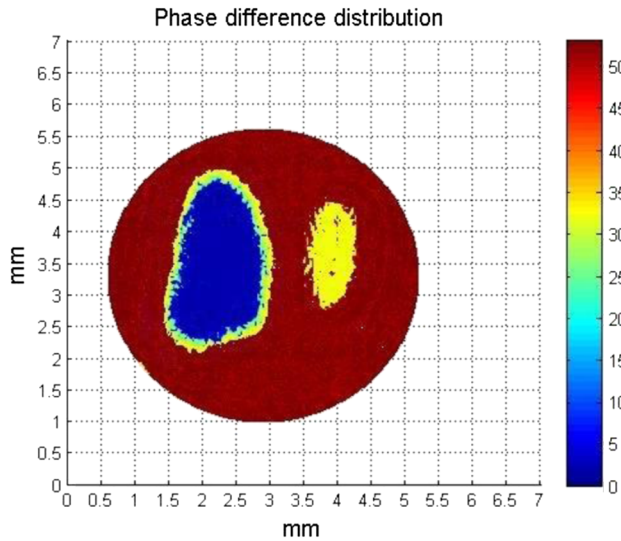


Fig. 3 The result of phase difference distribution.

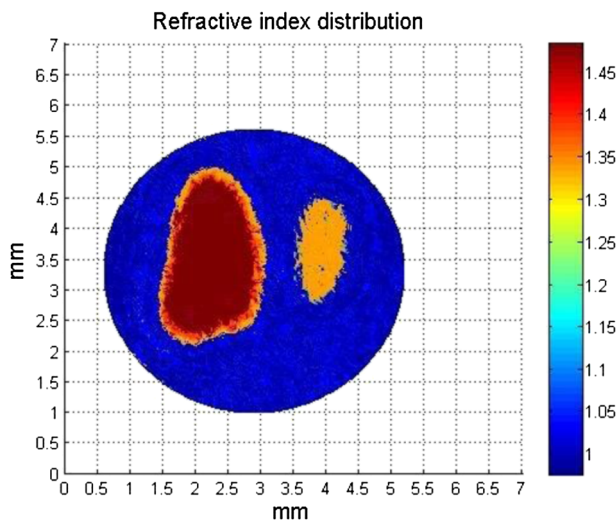


Fig. 4 The result of refractive index distribution.

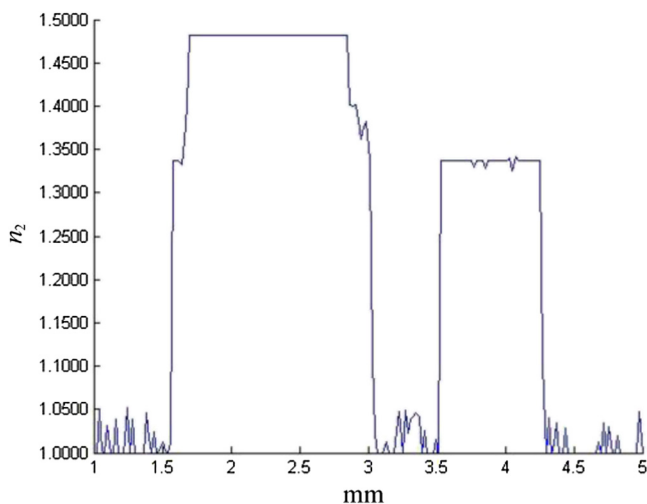


Fig. 5 The distribution of refractive index along the x-axis at  $y = 3.5$  mm.

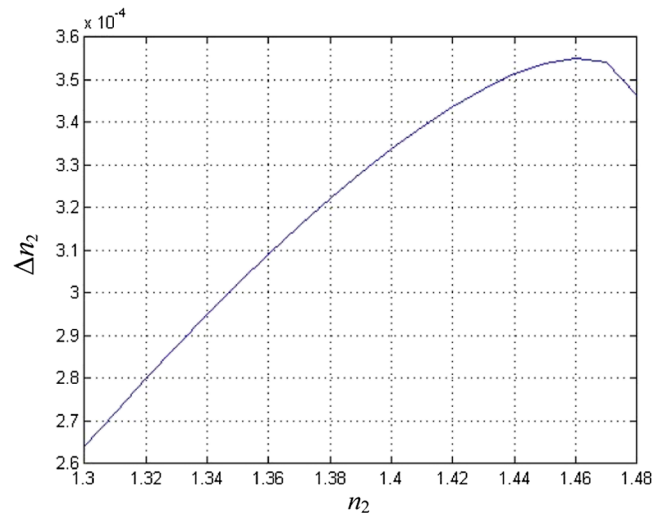


Fig. 6 Relationship of  $\Delta n_2$  versus  $n_2$ .

1.4595 deg. Accordingly, the phase-shifting modulated error was 0.0042 deg by substituting the error of EO modulator  $\Delta\Gamma$  into Eqs. (10) to (14). Consequently, the total phase error  $\Delta\phi$  calculated in our experiment was 0.0170 deg. In addition, considering the fabricated tolerance of the right-angle prism and the resolution of the rotational stage, the value of  $|\Delta\phi|$  was found to be 0.0195 deg. Substituting these values and the related experimental conditions into Eqs. (15) to (17), the relationship of the measurement resolution  $\Delta n_2$  and the refractive index  $n_2$  can be obtained, as shown in Fig. 6. Because of the simultaneous phase difference and phase errors in Carré phase-shifting algorithm, the curve appears as a nonlinear relation. The result in Fig. 6 displays that the measurement resolution has the highest value at  $3.552 \times 10^{-4}$  refractive index unit (RIU), when the refractive index  $n_2$  equals 1.460. Consequently, in our method, the measurement resolution can reach a value of at least  $3.552 \times 10^{-4}$  RIU.

#### 4 Conclusion

This paper proposes a simple method for measuring full-field distribution of refractive index. It is based on the phenomenon of total internal reflection and phase-shifting technique with modulated EO modulator. Experiments confirm the feasibility of this method. The measurement resolution can reach a value of at least  $3.552 \times 10^{-4}$  RIU. This method offers the benefits of simple installation, ease of operation, and rapid measurement.

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